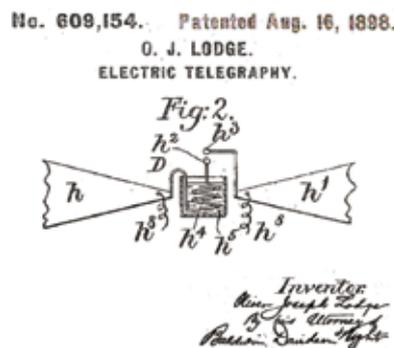


# Electromagnetic horns with elliptic plate separation

by Prof Jan Malherbe

The early development of electromagnetics involved the application of one of the most important contributions to modern science: the application of the vector differential equations, popularly (not always so popular with electrical engineering students) known as Maxwell's equations<sup>1</sup> in 1873.

Maxwell's theory had remained unverified until Heinrich Hertz conducted an experiment in 1887 that proved that the phenomena predicted by Maxwell did, in fact, exist. Experimental scientists the world over tackled the new development in science. When asked what the use of the new invention (radio) would be, Hertz is reported to have replied: "Nothing, I guess." In 1898, Lodge<sup>2</sup> described the radiating system shown in Figure 1 in a patent under the title of 'Electric Telegraphy'. Lodge claimed his patent for narrow bandwidth application ('tuned or timed or syntonised'<sup>2</sup>).



→ 1. Patent for radio system by Lodge<sup>2</sup>.

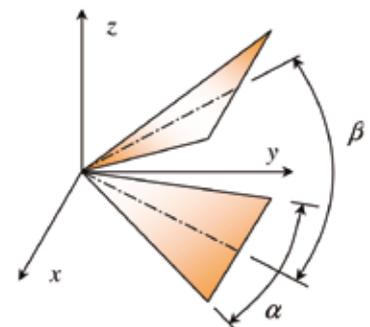
It is the first mention of a bowtie antenna. If the bowtie is folded at the apex, as shown in Figure 2, a structure results that lends itself to analysis in the spherical coordinate system. It will support a Transverse Electromagnetic (TEM) wave, and, providing the structure is infinitely long, will exhibit properties that are independent of frequency. In practice, the structure has to be truncated, and such TEM horns have a substantial but limited frequency bandwidth over which an acceptable match between the antenna and a source would be determined.

The improvement of the bandwidth properties became an issue with the introduction of television, multiple television stations, radar and other

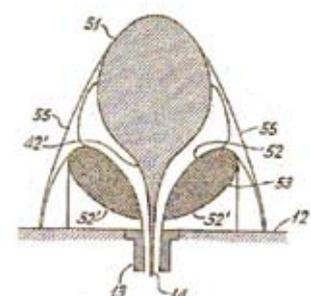
microwave systems. Lindenblad<sup>3</sup> patented a wideband antenna, shown in Figure 3, a prototype of which was mounted on top of the Empire State Building for many years. He claimed: "...more perfect impedance match... for much wider bands of frequencies..."

However, the first really wideband structures are the horns described by Brillouin<sup>4</sup> in 1948. The directional horn shown in Figure 4 anticipates the TEM structures that are described in this paper; the claims stated in his patent are shown in Table 2.

In spite of the obvious system advantages that an antenna designed according to the principles defined by Brillouin would have, it seems to have had very little impact on the development of antennas of very wide bandwidth. Presently, such structures find application in a variety of systems that are used in impulse-based applications such as ground-penetrating radar for mine detection, and so on.



→ 2. Basic TEM horn structure.

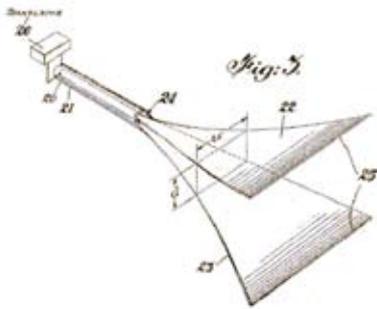


→ 3. Antenna by Lindenblad<sup>3</sup>.

**Table 1. Antenna terms**

|                                    |  |
|------------------------------------|--|
| frequency bandwidth                | The band of frequencies over which a specific parameter remains within a specified acceptable range.   |
| frequency-independent              | When a variable or variables are independent of frequency, it is said that a structure has frequency-independent properties.   |
| narrow band                        | If the band of frequencies over which a parameter varies is substantially limited, the device has a narrow bandwidth.  |
| broadband, wideband                | General term describing a device of which the parameters retain their acceptable properties over a wide range of frequencies.  |
| impedance match                    | The extent to which the impedance of a device matches that of a source or a load. If the device impedance equals that of the source or load, maximum power is transferred. |
| voltage standing wave ratio (VSWR) | Measure of the impedance match. If the VSWR=1, a perfect match is achieved; if VSWR<2, the match is considered good.   |
| gain                               | The extent to which an antenna can direct power in a given direction.<br>It is measured in decibels (dB) and is analogous to the gain of an amplifier.                     |
| E-plane                            | The plane containing the major electric field vector. For the horn of Figure 2, this would be the $z$ - $y$ (vertical) plane.  |
| H-plane                            | The plane containing the major magnetic field vector. For the horn of Figure 2, this would be the $x$ - $y$ (horizontal) plane.  |

In this paper, the development of a TEM horn antenna with extremely wide bandwidth, satisfying most of the objects of Brillouin's patent, is described.



→ 4. Directional wideband horn<sup>4</sup>.

Structure analysis and synthesis

The essence of an analytical approach to antenna design is a description of the physical phenomenon. A series of publications analysed the elemental structure of Figure 2, the most notable being the work of Lee and Smith<sup>5</sup>, who derived equations for the characteristic impedance of the simple horn. They showed that for practical designs, the equations for a microstrip transmission line over a ground plane can be used.

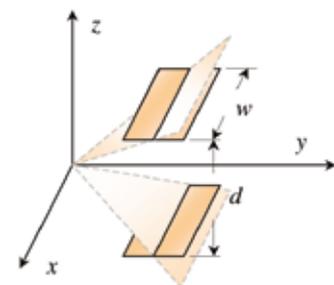
**Table 2. Brillouin's claims<sup>4</sup>**

|  |  |
|--|--|
| <p><b>An object of my invention is to provide a broad band antenna having a terminal impedance of a value approximating the value of the characteristic impedance of free space.</b></p> <p><b>Another object of my invention is to provide a radiating system which has a high reactance for a very broad frequency band extending over several harmonics.</b></p> <p><b>Another object of my invention is to provide a broad band antenna system which maintains substantially constant directivity over a broad band of frequencies.</b></p> <p><b>Another object of my invention is to provide an antenna system which is aperiodic in the sense that it has substantially no upper limiting frequency.</b></p> <p><b>Another object of my invention is to provide a transmission line and antenna system on which there are substantially no reflections due to abrupt variations in impedance.</b></p> | <p>5</p> <p>10</p> <p>15</p> <p>20</p> |
|--|--|

one succeeds in providing a smooth transition from the impedance of the feed line to that of free space. It is in order to achieve this smooth transition that a number of different design approaches have been developed, not all of which are based on sound argument.

Other approaches<sup>6</sup> make use of a simplified equation, using the properties of a parallel plate waveguide. For both these approaches, the horn is considered to be locally flat, as shown in Figure 5.

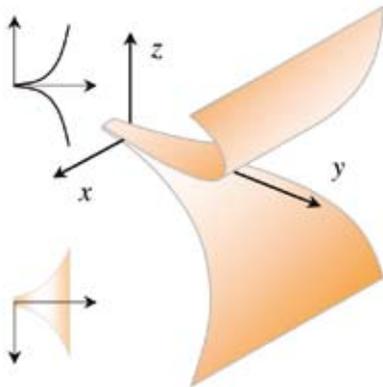
If we consider Brillouin's patent claims to be a specification, the horn must be designed to match the impedance of free space at the aperture (line 5 of his patent, Table 2). The other claims are almost automatically satisfied if



→ 5. Approximating the horn cross-section.

## Exponential taper

The simplest form of taper between two impedances is linear. However, this would obviously have two severe discontinuities, at the start and the end of the taper. A popular alternative, not based on sound premise, is to make use of an exponential taper. This presents a smooth transition at the feed point of the horn, but does not treat the aperture of the horn in any specific way. In Choi and Lee<sup>6</sup>, the characteristic impedance of the horn cross-section is made to vary exponentially between the feed point and free space characteristic impedance. As the horn impedance is determined by both the width,  $w$ , and plate separation,  $d$ , an additional constraint needs to be introduced, and the plate separation is chosen to be exponential.



→ 6. TEM horn with exponential impedance taper and exponential variation of plate separation<sup>6</sup>.

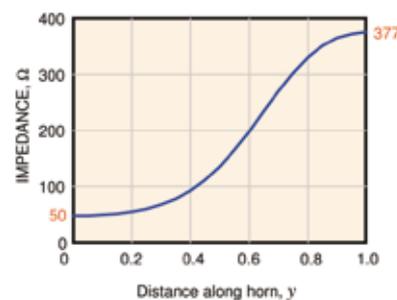
The horn is shown in Figure 6, and resembles that described in Figure 4. The physical prototype of the horn is well matched ( $VSWR < 2$ ) over the frequency band 200 MHz to 1.2 GHz, or a bandwidth ratio of 6:1. A variety of other designs, employing similar principles, have been described; some of them improve the impedance match by adding radio-absorbing material at strategic places.

## Optimal impedance taper

Methods whereby two impedances can be matched arbitrarily well over a chosen bandwidth have evolved from the modern network theory. The near optimal taper of Hecken<sup>7</sup> is given by:

$$G(B, \xi) = \frac{B}{\sinh(B)} \int_0^{\xi} \left\{ B \sqrt{1 - \xi'^2} \right\} d\xi'$$

Figure 7 shows the variation of impedance along the length of a horn that has been designed to have an optimal match between feed and free space.

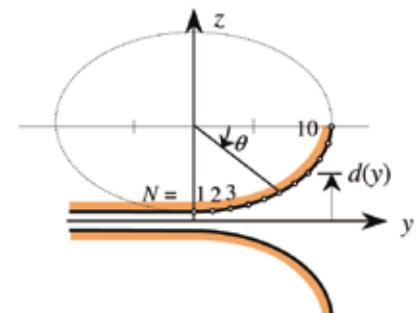


→ 7. Variation of impedance according to the near-optimal impedance function by Hecken<sup>7</sup>.

## Elliptic plate separation

Traditionally, designers have concentrated on either improving the matching function, or the conversion of impedance to dimensions by means of the known functions. Impedance, however, is not a physical parameter. It is only a ratio between two physical parameters (in this case, the voltage between the two plates and the current flowing along the plate) and therefore does not physically describe what is happening. The concept is useful in ensuring that large reflections do not occur because of sudden discontinuities in voltage or current. However, the one aspect not addressed is the fact that, in addition to voltage and current, there also needs to be a transition between the (bound) TEM mode

in the region of the horn that is an antenna, and the region of the horn (in or near the aperture) where the energy is radiated; the radiated mode is completely different from the bound TEM mode. It was therefore argued that the separation profile of the horn plates need to be totally smooth at the feed point, while curving sufficiently to present a surface normal to the electric field at the aperture. A section of one quarter of an ellipse, as shown in Figure 8, fulfils this requirement, and this becomes the basic property of a family of TEM horns.



→ 8. Elliptic plate separation profile.

Combined with the near-optimal impedance taper, all that is needed is a function to calculate the width  $d$  from the impedance.

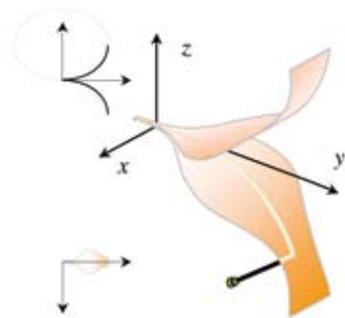
## Horn with microstrip impedance calculation

As a first attempt, a horn was designed with elliptic plate separation, near-optimal taper, and making use of the equations of Lee and Smith<sup>5</sup>, which are undoubtedly founded on a scientifically better argument than the parallel plate equations used in Choi and Lee<sup>6</sup>. Figure 9 shows the physical layout of the horn, which is almost startling. Because the impedance at the aperture attempts to approximate that of free space,  $\approx 377 \Omega$ , and the aperture height is determined by the height of the ellipse, the horn width is small.

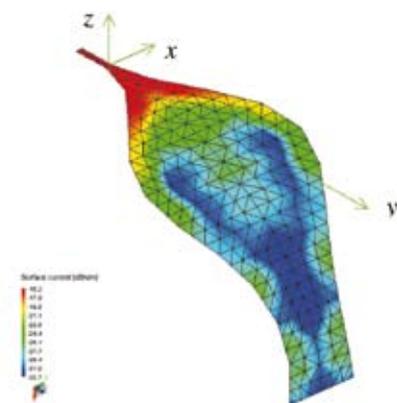
A prototype horn was constructed in brass plate, with the horn profile

maintained by means of dielectric pillars. In order to be comparable to the horn described in Choi and Lee<sup>6</sup>, the horn length and aperture height was chosen to be the same as that of Choi and Lee<sup>6</sup>, i.e. some 480 mm long and 732 mm high at the aperture.

The plates were only 6 mm wide and separated by 2 mm at the feed point, and in order to be able to feed the balanced horn by means of a coaxial cable, a null was identified on the outer surface of the horn by means of the commercial electromagnetic code, FEKO<sup>8</sup>. The surface currents on the outside of the horn are shown in Figure 10, where red denotes a high current density and blue a low current density. The VSWR of the prototype horn remained below 2 between 400 MHz and 6.2 GHz, or a bandwidth ratio of 15:1<sup>9</sup>.



→ 9. TEM horn with elliptical plate separation and microstrip impedance approximation.



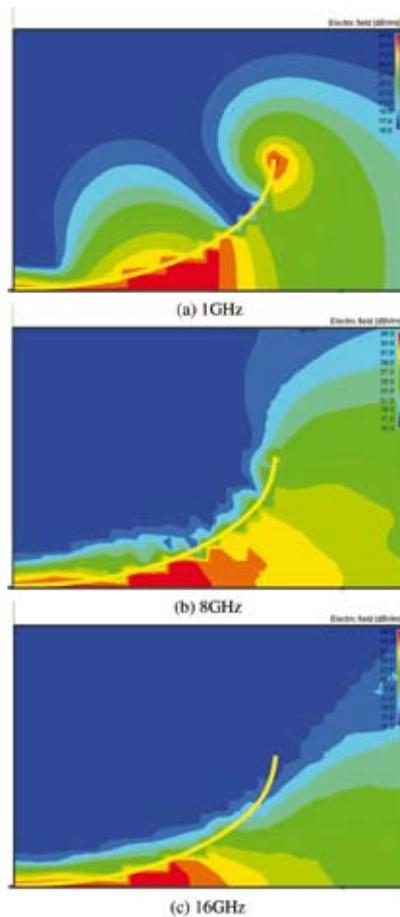
→ 10. Current density on the outside of horn plate surface. Only one plate is shown.

### Modified parallel plate approach

The calculation of the horn profile by means of the parallel plate waveguide equations yielded an entirely different profile to the microstrip approximation. The differences were so dramatic that the nature of the fields in the immediate vicinity were first examined in an effort to understand the mechanism of radiation.

Figure 11 shows the electric near fields of the horn, as calculated by FEKO at frequencies of 1, 8, and 16 GHz, respectively. In Figure 11a it is clear that at the low frequency, a resonance occurs in the electric field, showing strong peaks and nulls in field strength.

This is in accordance with the understanding that at the low



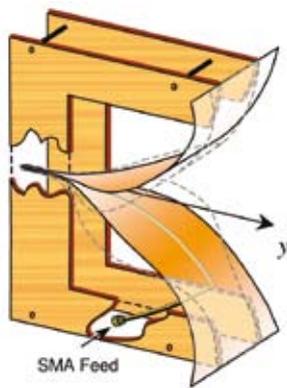
→ 11. Electric near fields of the horn.

frequency end, the horn in effect behaves like a resonant dipole. As the frequency is increased, as indicated in Figure 11b, the peaks and troughs begin to disappear, except for the region closer to the aperture of the horn. In the region closer to the feed point, the field amplitude remains constant. This signifies the existence of a travelling wave, which sheds energy while power is radiated towards the aperture of the horn.

Further increase in frequency leads to only the travelling wave phenomenon, and also another observation. Towards the aperture of the antenna (Figure 11c), the field amplitude decreases dramatically, indicating that, as the frequency is increased, radiation occurs increasingly from deeper in the 'throat' of the horn. This leads to the conclusion that the horn is in effect a high-pass structure, i.e. it would not exhibit a high-frequency cut-off. This aspect will be discussed again later.

Clearly, in the light of this insight, to speak of impedance and impedance functions becomes somewhat irrelevant. The impedance approach is only relevant as long as there are no standing waves, and no radiation takes place, which is a contradiction in principle. It does, however, seem to yield a gradual enough change in horn width so as to prevent unnecessary reflections.

The parallel plate waveguide approach would result in a horn of the same width as that in Choi and Lee<sup>6</sup> at the aperture; this was considered too wide, as it could lead to transverse modes along the edge of the aperture. At the same time, the broad flare halfway along the horn length could be problematic. Consequently, a horn was designed using one half of the width calculated for the parallel plate waveguide, and with an elliptical plate separation. The way in which the width function varied was still based on the Hecken near-optimal taper.

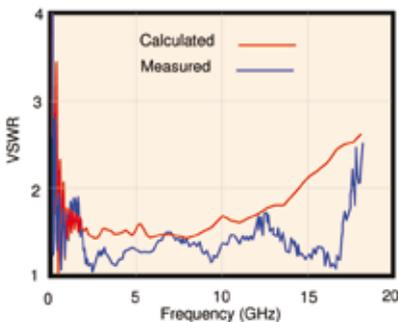


→ 12. *Extreme performance horn.*

The extreme-performance horn<sup>10</sup>

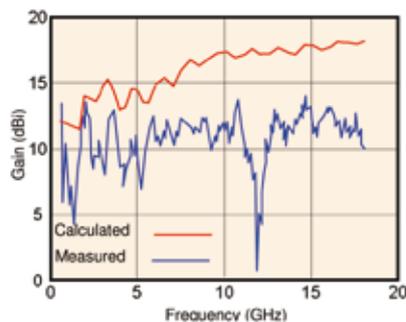
This design was termed the extreme-performance horn. The structure is shown in Figure 12, including the way in which the thin brass foils were mounted in a plywood structure.

The horn was also fed to a surface current null below one plate, as before. Shown ghosted on the horn profile of Figure 12 is the outline of the previous horn. The substantial differences in shape are obvious. Figure 13 shows the measured and calculated VSWR responses of the extreme performance horn. The reason for the choice of name is obvious. While the calculated VSWR bandwidth ran from 400 MHz to 14 GHz, or 35:1, the measured bandwidth extended from 250 MHz to 17.5 GHz, or a ratio of 70:1.



→ 13. *Calculated and measured values of the Voltage Standing Wave Ratio vs frequency for the extreme performance horn.*

Similarly, the response of gain vs frequency is shown in Figure 14. While the VSWR response is excellent, the gain response is somewhat disappointing, exhibiting a sharp drop in gain around 12 GHz. This is quite unexpected, and out of character for the structure.



→ 14. *Calculated and measured values of the gain vs frequency for the extreme performance horn.*

It must be borne in mind that the constructed prototype is of relatively flimsy structure. Bear in mind that the aperture height is 732 mm, while the plates are only 2 mm apart at the feed point; at the aperture the horn is 366 mm wide, and only 6 mm at the feed point. The elliptical shape is maintained only by the initial bending of the plates, and it is possible that this differs substantially from the desired shape on a point-by-point basis.

It is therefore not only conceivable, but also probable that at specific frequencies, higher-order modes would be generated because of the deviation from the desired values. These modes would radiate in arbitrary directions, and thereby cause the type of drop in gain that was observed.

The numerically calculated VSWR rises smoothly beyond about 8 GHz. Evaluation of the impedance shows that the imaginary part exhibits exactly the same trend, while the real part remains constant. The addition of a small metal tab at the point where the horn is fed cancels a substantial

part of this unwanted reactance, accounting for the substantial increase in bandwidth.

Calibrating numerical calculation to measurement

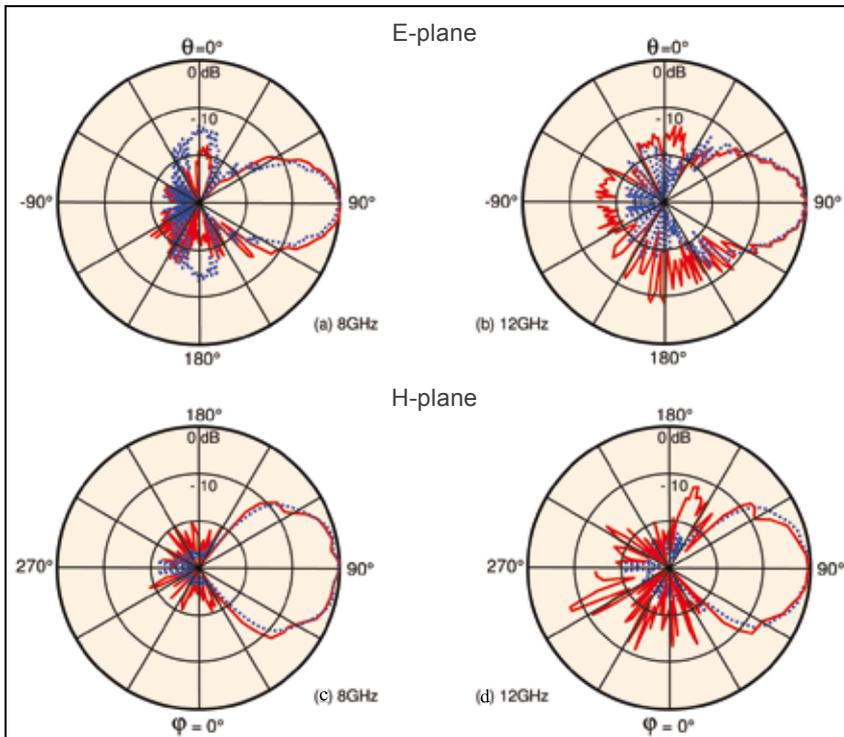
In order to further develop the elliptical separation horn, it was necessary to ensure that there was reliable correspondence between the numerical calculations using FEKO and the measurements on the physical model.

Consequently, extensive measurements were made on the prototype horn, and compared to calculations. The radiation patterns were measured for both the E-plane and H-plane at frequencies of 0.5, 1, 2, 4, 8, 12, 15 and 18 GHz and compared to the calculated values<sup>11</sup>. Examples of these results are given for 8 and 12 GHz in Figure 15. They showed that there was excellent agreement over the main lobes of all the patterns, and that the sidelobe performance was within acceptable limits. This, in turn, implied that a next step of predicting horn performance based on calculations could be done with reasonable confidence.

Frequency-independent properties

A parametric study of the extreme-performance horn was performed in order to establish the useful range of structures. Note that electromagnetic wave structures scale with frequency; that is, if we halve the physical size of a structure, the performance against frequency will be repeated in a band of frequencies double that of the original structure, and vice versa.

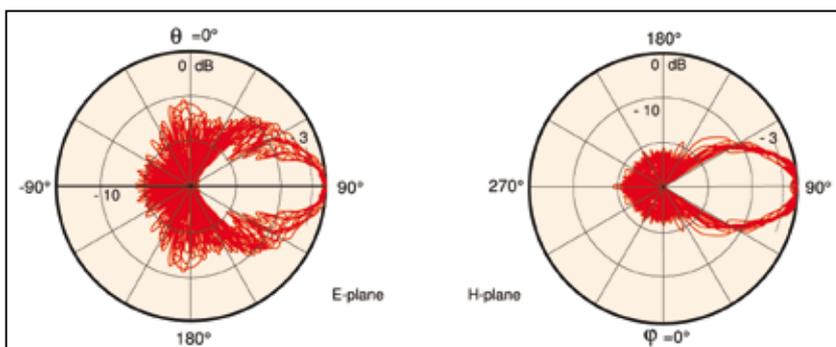
The parametric study then comprised the evaluation of four horns with elliptical plate separation, and ratios of aperture height to lengths of 0.5, 1.0, 1.525 and 2.0, respectively. This corresponds to aperture heights of 240, 480, 732 and 960 mm respectively, while all the horns



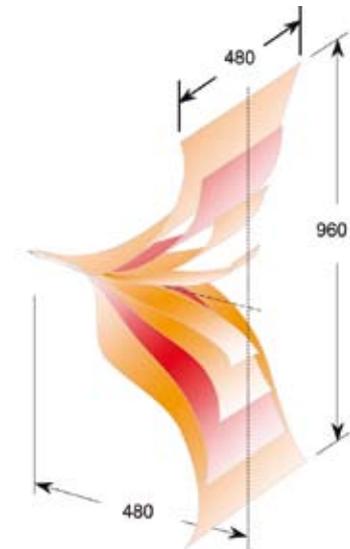
→ 15. Comparison of measured and calculated E-plane ((a) and (b)) and H-plane ((c) and (d)) at 8GHz ((a) and (c)) and 12 GHz ((b) and (d)) for the extreme bandwidth horn.

were 480 mm long. Figure 16 shows a comparison of the series of horns.

Radiation patterns were calculated in both major planes, at frequencies from 1 to 18 GHz, with 1 GHz intervals<sup>10</sup>. For the horn with an aspect ratio of 1, the radiation patterns were virtually stable over the entire frequency band (Figure 17) where all 18 patterns are plotted on one polar diagram.



→ 17. Grouped radiation patterns at 1 GHz intervals for various horns, E- and H-planes.



→ 16. Comparison of horns of various aspect ratio. The extreme performance horn is in red.

### Conclusion

Returning to the patent claims that Brillouin had lodged in 1948, the present development of the elliptic horn satisfies that specification to all intents and purposes. It is indeed an extremely wideband antenna, with excellent matching to the feed impedance, constant radiation pattern, and high gain.

For further development of the antenna as a high-pass structure, it would be necessary to pay extreme care to the point at which it is fed. With increasing frequency, the region in the vicinity of the feed point becomes more important dimensionally. The implication is that both the width and plate separation would become increasingly smaller, until it becomes impractical to construct the antenna, because, among other reasons, at high power levels such small gaps would give rise to arcing.

A practical solution to the construction problem would be to form a dielectric shape of very low dielectric constant, such as expanded polystyrene to a high degree of accuracy, and then to lay down a conducting surface of foil over it, to form the horn. ☺

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