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Extreme-angle broadband metamaterial lens

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For centuries, the conventional approach to lens design has been to grind the surfaces of a uniform material in such a manner as to sculpt the paths that rays of light follow as they transit through the interfaces. Refractive lenses formed by this procedure of bending the surfaces can be of extremely high quality, but are nevertheless limited by geometrical and wave aberrations that are inherent to the manner in which light refracts at the interface between two materials. Conceptually, a more natural—but usually less convenient—approach to lens design would be to vary the refractive index throughout an entire volume of space. In this manner, far greater control can be achieved over the ray trajectories. Here, we demonstrate how powerful emerging techniques in the field of transformation optics can be used to harness the flexibility of gradient index materials for imaging applications. In particular we design and experimentally demonstrate a lens that is broadband (more than a full decade bandwidth), has a field-of-view approaching 180° and zero *f*-number. Measurements on a metamaterial implementation of the lens illustrate the practicality of transformation optics to achieve a new class of optical devices.

As the trajectory of light is altered only at the input and the output surfaces of a conventional lens—and left to travel in a straight line within the volume of the lens—it is difficult to create as ideal an optical device as one would desire. Monochromatic aberrations, such as spherical or coma, are generally unavoidable with refractive optics and can at best be minimized through the use of systems of many lenses. The aberration profiles of lenses place an ultimate limit on certain high-performance imaging applications. Wide-angle imaging systems, for example, make use of stacks of lenses, yet often show significant distortion even after optimization.

Gradient index (GRIN) lenses represent an alternative approach to lens design. Rather than relying on the interfaces of a uniform material to manipulate light, the index of refraction is varied throughout the body of the lens. Rays are no longer abandoned once entering the medium, but instead can be guided with far greater control to their ultimate destination. Although GRIN lenses have significant potential advantages over conventional lenses, they are far less prevalent in practical applications because the process of achieving large index gradients in a controlled manner poses a difficult fabrication challenge. The advent of metamaterials coupled with nanoscale lithographies, however, suggests that GRIN optical elements may be much more feasible using artificially structured media[1]. Moreover, as metamaterials offer a significantly broader range of material properties, including both electric and magnetic response and anisotropy, we would expect a correspondingly broader range of potential lens designs.

Indeed, the large phase space associated with metamaterials does provide greater opportunities for lens design, but brings with it the need for tools that can efficiently provide a design once a given functionality is specified. The recently reported technique of transformation optics provides just such a tool. Transformation optics is a means to visualize the interaction of a class of complex materials with electromagnetic fields in terms of a warping of space[2–6]. Transformation optics has been used to design conceptually new and creative devices for conventional electromagnetic applications such as beam shifters, splitters[7], collimators[8] and even lenses[9–11]. Transformation optics has also been used in the design of new classes of devices that inspire the imagination such as electromagnetic cloaks[12–17] and wormholes[18].

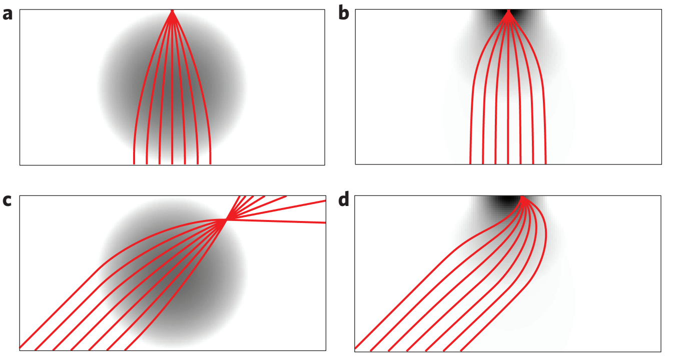

Figure 1 | Ray tracing results for spherical and flattened Luneburg lenses. Regions of refractive index of less than one in the flattened Luneburg lens are not included in the index profiles used for these ray traces following the approximation described in the text. **a**,**b**, Strong focusing of rays normally incident on a Luneburg lens and the flattened Luneburg lens presented here. **c**, In a Luneburg lens, rays at oblique incidence are focused onto a spherical surface. **d**, With the modified index profile, even rays at extreme angles are focused onto the image plane.

To illustrate the use of recently developed transformation optics approaches in the context of lens design, we start with a particular optical element and apply a transformation to improve the form factor and overall utility of the optic. It has long been known that it is possible to create a spherical lens with no aberrations for which the locus of focal points resides on a sphere[19]. Such a lens is called a Luneburg lens after the inventor. These lenses have found some success in commercial applications, but are heavily restricted from broad use in imaging tools for two reasons. The first is that they are manifestly gradient-index devices, which are more difficult to manufacture. The second is that the spherical locus of focal points represents an inherent mismatch to conventional detector/receiver arrays, which are generally planar.

Nascent tools in the field of metamaterials suggest that both of these obstacles can be overcome. The gradient-index requirement can be addressed by the use of patterned metallic or metallodielectric inclusions. The refractive index of this medium can be found using metamaterial-homogenization techniques[20]. The more fundamental problem with Luneburg and similar

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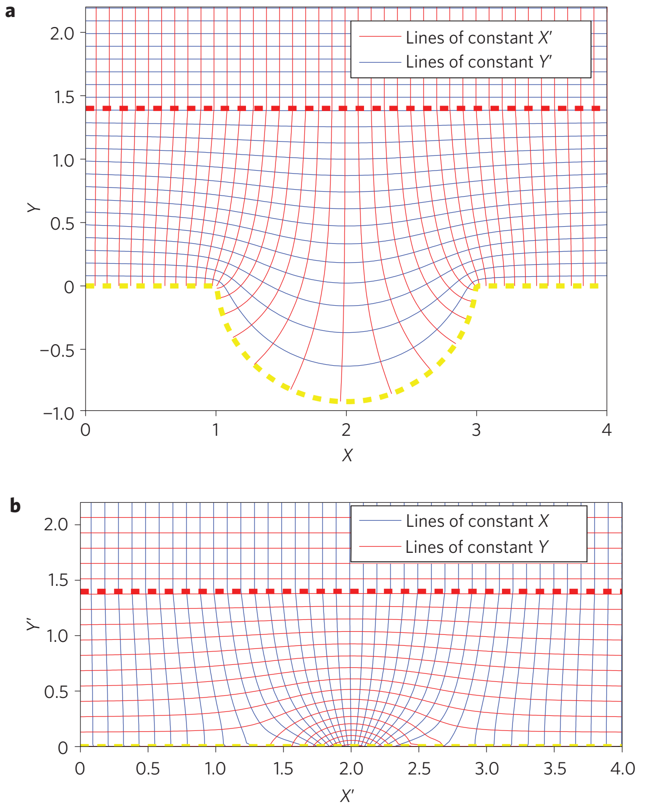

Figure 2 | The transformation used to produce the flattened Luneburg lens. The material used to realize the flattened Luneburg was designed with a numerically generated quasi-conformal spatial transformation. **a**,**b**, The resulting map (**a**) and its inverse (**b**). Materials designed in this manner are inherently broadband, as they require only dielectric materials.

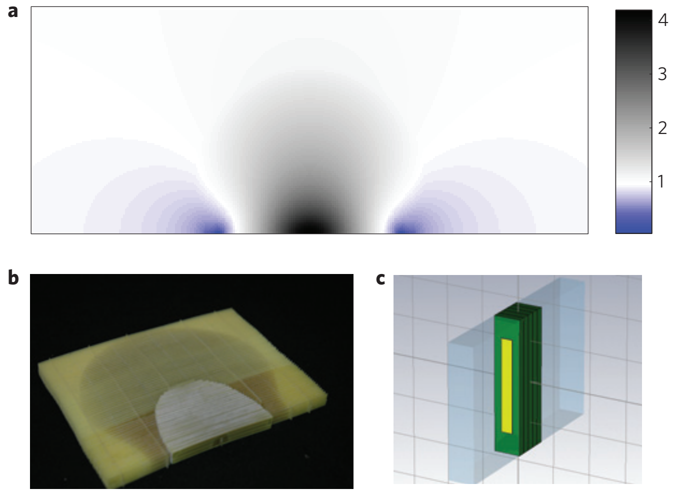

Figure 3 | Lens design and fabrication. a, The index profile used in the realization of the flattened Luneburg lens. Regions of $n > 1$ are shown in black. Regions of $n < 1$ are shown in blue. The regions of $n < 1$ were neglected in the experimental realization of the lens. This approximation is relatively benign for all but the highest incidence angles because the fields are focused in the high-index region. **b**, The lens was broken into two regions. The outside layer contained the low-index ($n < 2$) material and the dense inner region contained the high-index ($2 < n < 4.1$) material. **c**, Copper strips on an FR4 substrate were used as polarizable particles to realize the index distribution.

lenses—that the spherical focal locus is inherently unsuitable for standard CCD (charge-coupled device) or antenna arrays—can be addressed by flattening the focal plane through the application of transformation optics[21]. Applying the extra technique of quasi-conformal mapping, solutions can be found that do not require the use of resonant metamaterials, implying that the resulting optic can have a large bandwidth.

Transformation optics takes advantage of the fact that Maxwell's equations can be written such that they are 'form invariant' under coordinate transformations[5]. Under such coordinate transformations, the material parameters transform as second-rank tensors of weight +1(ref. 4), or

$$\varepsilon_r^{i'j'} = \frac{1}{|A|} A_i^{i'} A_j^{j'} \varepsilon_r^{ij}$$

$$\mu_r^{i'j'} = \frac{1}{|A|} A_i^{i'} A_j^{j'} \mu_r^{ij}$$

where

$$A_i^{i'} = \frac{\partial x^{i'}}{\partial x^i}$$

A transformation optics approach to the warping of a Luneburg lens has previously been considered[21]. The idea behind the transformation is to take a region with a spherical protrusion and flatten it. The transformation does not change the nature of the original lens as a perfect imaging system; only the shape of the focal plane is changed.

In both ref. 21 and the present approach, the transformation carried out is not extended past the focal plane of the lens, which is assumed to be a caustic surface. As the transformation is thus embedded in free space, the effect of the transformation will not be invisible to an observer, as is normally the case with transformation optics media[7]. It is assumed that the lens will be terminated with an imaging or antenna array.

The implementation of the transformations studied in ref. 21 requires strongly anisotropic materials, leading to the necessary inclusion of resonant particles and their inherent bandwidth limitations and loss. In the present approach, these limitations are overcome, resulting in a lens that can be naturally scaled to infrared or even visible wavelengths.

For the development of flattened Luneburg lens, we have used the recently introduced technique of quasi-conformal transformation optics (QCTO), which for some transformations can result in a device with identical functionality to a standard transformation-optics design, but can be implemented using isotropic materials relying only on gradients in the refractive index of the medium[22]. Ray tracing results that contrast the focusing properties of the flattened lens design using QCTO with the original index profile of the Luneburg lens are shown in Fig. 1. This technique has been demonstrated experimentally as a broadband 'carpet-cloak' at both microwave[23] and optical frequencies[24,25].

The QCTO design method makes use of a set of boundary conditions that define the effect of the device, while restricting the transform inside the material. In particular, angles between the coordinate lines are approximately preserved and there is limited anisotropy of the coordinate 'squares' (that is, they are still approximately square, rather than rectangular). In contrast with optical conformal mapping[13], the QCTO technique allows the conformal module of the two domains to differ to a limited extent—a crucial feature for our design. For a two-dimensional transform, this leads to the prescription of a material with very limited anisotropy that can be well approximated by a dielectric-only response. Standard conformal grid generation methods may be used to realize such a transformation[22].

The quasi-conformal map for the flattening of a portion of a spherical lens is shown in Fig. 2. The circular protrusion that follows

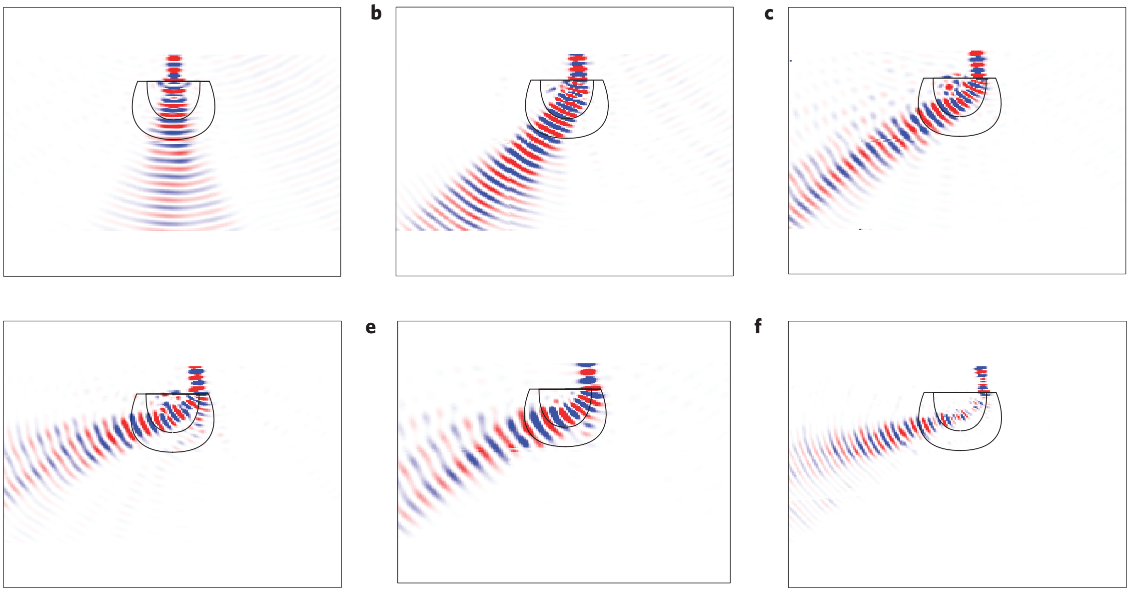

Figure 4 | Experimental field maps of the flattened lens. The electric field is shown on an arbitrary scale. **a–d**, Beams are directed at 0° (**a**), 35° (**b**), 50° (**c**) and 70° (**d**) from normal at 10 GHz. To produce these beams, a dielectric waveguide was used to introduce an effective point source excitation at different positions along the back of the lens. The lens turns these sources into high-directivity waves travelling in different directions. Note that, unlike many metamaterial structures, this device is broadband and shows very low loss. **e**,**f**, Images taken at 7 GHz (**e**) and 15 GHz (**f**) demonstrating the broadband nature of the lens.

the yellow line is mapped to a straight line that will make up the focal plane of the lens. Slipping boundary conditions are used along the yellow line in the quasi-conformal relaxation procedure. The transformation is ended with a Dirichlet boundary condition at the red line. The choice of $y = 1.4$ for this boundary is arbitrary; in fact, the boundary is not required to be a flat line at all. However, if the transformed region is too small, the conformal module will change significantly and the approximation of an isotropic medium will fail.

The Luneburg quasi-conformal transformation is slightly different in concept from the one carried out recently for the ground-plane (or 'carpet') cloak[22]. For the final design of the Luneburg, it is desired that a flat plane will behave as though it were warped around the back of the lens; for the carpet cloak, a warped plane is made to behave as though it were flat. Although the underlying techniques are identical for the two configurations, we make use of the inverse map here, which is also quasi-conformal, to realize the transformation. Both the original quasi-conformal map and its inverse are shown in Fig. 2.

The lens fabricated to demonstrate the flattened Luneburg design was built to work in a parallel-plate waveguide that restricts the electromagnetic fields to two-dimensional transverse-electric-polarized waves. Two-dimensional systems allow for the experimental mapping of the fields within the lenses using sensitive near-field probes and a phase-sensitive network analyser. This measurement apparatus has been previously described[12].

The index profile required to produce this lens is shown in Fig. 3a. Some approximations were necessary in the experimental realization of the lens, however. In particular the index profile had to be adjusted to remove regions of $n < 1$. Although an index of less than 1 is achievable using metamaterials, such a device would be inherently dispersive, which would in turn limit the bandwidth. To side-step this limitation, the index was set to 1 in any region where it had previously been less than 1. Although this approximation is crude, its effect is relatively benign. Full-wave simulations and ray tracing results of devices containing the $n < 1$ regions showed similar results to those that did not. This result is qualitatively attributed to the fact that the light is strongly focused to the high-index regions of the lens, largely avoiding the low-index regions for all but the highest incidence angles.

An index range of 1.08–4.1 was used in the implementation of this design. The index profile was achieved by patterning copper strips on an FR4 substrate. The copper strips act as polarizable inclusions in the medium, increasing the overall polarizability of the material. These strips are not resonant in the frequency range of operation for the lens, and simulations indicate that the dissipative loss of these metamaterial cells is essentially equal to that of the substrate material. To achieve the requisite index range, the lens was divided into two regions. The first contained the regions of materials with an index between 1 and 2, and was fabricated in a similar manner to previous metamaterial GRIN devices[23]. The second region had an index range from 2 to 4.1 and was composed of 220-μm-thick FR4 and copper strips laid directly on top of one another. This compact design allowed the index to be brought to over four without using resonant particles or introducing excessive spatial dispersion—both of which would have imposed bandwidth limitations. In the band from 7 to 15 GHz the index changes by less than 10% in the region with the largest dispersion (the highest index region), and less than 3% in most of the device.

A discrete set of bars with defined lengths were simulated to find their effective index using a standard retrieval method[20]. An interpolation scheme was then used to find the appropriate bar length for the desired index at each point in the structure. The completed device is shown in Fig. 3b.

The lens was tested using a dielectric waveguide to apply a source to different positions along the back surface of the lens. This set-up allowed the effects of the impedance mismatch on the back of the lens to be minimized. The lens produces approximate plane waves propagating in different directions as the position of the source is varied—extending to extreme angles approaching the horizon. The resulting field profiles are shown in Fig. 4. Note that the material response is broadband. The effect of the lens does not qualitatively change between 7 and 15 GHz, the frequency range of our measurement capabilities.

We have designed and demonstrated a single-element wide-angle lens with a field of view approaching 180°. The lens has a flat focal locus, making it usable with standard imaging arrays. Furthermore, no resonant particles were required in the fabrication of the lens, resulting in a device that is broadband and shows limited loss. In this case, the loss is essentially equal to that of the substrate material. Experiments were carried out in the X-band microwave regime and showed identical behaviour from 7 to 15 GHz, the full spectrum of our measurement system. We expect that such a lens may be useful in telecommunications or radar applications where a wide field of view and high gain are needed. The lack of resonant particles, however, makes scaling to infrared or even optical frequencies possible. The primary drawback of this approach is that orthogonal mappings are generally restricted to two dimensions. However, the lens design discussed here may provide a good initial step in developing a three-dimensional lens.

These results are particularly compelling given the relative simplicity of the experiment. This design may be of practical importance, but it also serves to underscore the fundamentally new devices that can be produced when control of the volume of a material is used. We expect that the techniques used in the design of this lens will be of importance in many areas of electromagnetic design. At short wavelengths, they may be used to design replacement optical devices that have previously required the use of complex systems of many lenses. At longer wavelengths such systems are impractical because of the size and weight limitations, thus making the lens we have designed a fundamentally new tool. At microwave frequencies, for example, a wide field of view can be achieved only by using moving parts or complex systems such as phased-array antennas or a-planar antenna arrays—devices with significant bandwidth limitations.

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Author contributions

N.K. and D.R.S. jointly conceived the strategy of leveraging QCTO for the lens flattening procedure. N.K. conceived of using the inverse transform, implemented a relaxation method to carry out the transform, designed the metamaterial lens and characterized the lens though simulations, ray-tracing and experiment. D.R.S. supervised the design and execution of the experiments. The manuscript was prepared by N.K. in collaboration with D.R.S.

Additional information

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