

# Non-Foster Augmentation of a Multiband LMR Antenna System

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June 30, 2012

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# 1 Introduction

This report describes a possible non-Foster augmentation for a multiband land mobile radio (LMR) antenna system being developed under our project “Antenna Systems for Multiband Mobile & Portable Radio” [1]. The antenna system is intended for use in multiband vehicle-mounted push-to-talk systems, such as those commonly employed in public safety organizations, operating at frequencies ranging from about 25 MHz to about 900 MHz in 5 bands. The most recent version of the antenna system is known as the “Phase II” design, which is documented in [2] and references therein. This design consists a reconfigurable (two-state) monopole with a separate electronic tuner that selects the state of the reconfigurable monopole and automatically matches the antenna impedance using an internal network of tuning stubs. As explained in Section 2, the benefit of Non-Foster technology in systems such as this is the ability to achieve larger instantaneous bandwidths and/or smaller antennas. The benefit of larger instantaneous bandwidth is the ability to effectively receive signals over a larger frequency range simultaneously; that is, without reconfiguring the antenna or changing the tuning solution implemented within the tuner. Based on these considerations, Section 3 describes what appears to be the most suitable and realistic application for Non-Foster technology for our antenna system: Eliminating the low-frequency extension of the reconfigurable monopole and broadbanding the performance in the VHF-Low (25–50 MHz) band. Section 4 explores the practical requirements for a non-Foster solution to this problem. Section 5 then describes a preliminary design and shows the expected performance.

## 2 Introduction to Non-Foster Technology

Foster’s Theorem states that *the first derivative with respect to frequency of the reactance of a passive lossless two-port circuit is always positive*. Specifically, the reactance of lossless (but otherwise physically-realizable) capacitors and inductors, and of circuits containing any combination of these devices, always increases with increasing frequency. As it turns out, the reactance of electrically-small antennas is negative (“capacitive”), very large, and also increases with increasing frequency. A perfect single-frequency match to an electrically-small antenna is always possible in principle, since one can always specify a series inductor whose negative reactance exactly cancels the antenna’s capacitive reactance at one frequency, and then a transformer can be used to match the real part of the antenna’s impedance without introducing additional reactance. However the reactances of both the antenna and the series inductor increase with frequency, thus the match quickly “unravels” away from the nominal frequency. What is needed for a truly broadband match is not an inductor, but some other device which has also positive reactance but which *decreases* with frequency in the same manner that the antenna’s reactance increases with frequency. A “negative capacitor” – a device whose capacitance is opposite in sign to that of a conventional capacitor – would have these characteristics, but is physically unrealizable as it would violate Foster’s Theorem. Similarly, no other passive lossless device or combination of such devices can have this characteristic.

A “non-Foster” device (or circuit) is one which exhibits reactance which decreases with frequency; for example, the negative capacitor described above. Such a circuit is physically realizable only if it is *active*; that is, non-Foster behavior requires a source of power distinct from the signal of interest. Thus, non-Foster devices typically involve devices such as transistors or operational amplifiers that can apply gain to signals. An important class of non-Foster circuits are *negative impedance converters* (NICs), which change the sign of the impedance of physical devices and circuits – e.g., transforming a capacitor into a negative capacitor, an inductor into a negative inductor, and (generally) transforming the impedance of a circuit with impedance  $Z(f)$  (where  $f$  is frequency) into  $-Z(f)$ . A seminal discussion of NIC circuit theory appears in Linvill (1953) [3]. An early demonstration of NIC-implemented negative capacitance to the broadbanding of electrically-small antennas appears in Bahr (1977) [4], who demonstrated the principle for a short monopole operating in 30–60 MHz, but encountered stability and noise problems which made the approach impractical. The concept has in the last decade been revived and demonstrated by others with somewhat greater success; see in particular Sussman-Fort (2006) [5] and Sussman-Fort & Rudish (2009) [6]. The latter is notable

as it includes a reproducible NIC design for a 15 cm monopole which appears to be effective in 20–120 MHz. Thus, this design might be applicable to the 23-cm monopole used in the Phase I design and in the “base element” of the Phase II design operating in the VHF-Low (25–50 MHz) band. This idea is pursued further in Sections 3–5.

Before proceeding, it is appropriate to consider what other applications non-Foster technology might have in improving the capabilities of the Phase II antenna system. In principle, the ability to employ non-Foster impedances as antenna matching elements removes limitations on the achievable bandwidth of matching. These limitations are quantified in the Fano-Bode relationships [7] (see also the much more readable [8]), which constrain the frequency range over which the reflection coefficient at the antenna terminals can be better than a given specification. One typically finds that for “thin” monopoles longer than about one-fifth of a wavelength with passive matching networks, voltage standing wave ratios (VSWRs)  $\leq 2$  are possible over bandwidths less than  $\approx 5\%$  at best. For electrically-short monopoles, the situation is *much* worse, as will be demonstrated in Section 3. Thus, an obvious application for non-Foster matching in the present project is to extend the instantaneous impedance bandwidth to cover entire bands simultaneously, which is not generally possible with the stub-tuning approach employed by the Phase II tuner. However all recent investigators (including the authors of [5] and [6]) have noted the extreme difficulty in implementation of NIC circuits above 100 MHz. Due to the limited time and resources remaining in this project, we choose to focus on the VHF-Low (25–50 MHz) band, where the existing technology seems most likely to be successful.

A non-Foster match in the VHF-Low band for the reconfigurable monopole in either state could be expected to allow the instantaneous impedance bandwidth to be increased, potentially covering the entire band and removing the need to tune within this band. A non-Foster match for the reconfigurable monopole in the *high*-band state (i.e., using only the 23 cm-long base element) that covers the entire VHF-Low band (25–50 MHz) might render the extension element unnecessary, resulting in a much more compact antenna that is also simpler in that it does not need to be reconfigurable. For this reason we will focus on the possibility of an acceptable VHF-Low receive capability using only the 23 cm base element from the Phase II design, using non-Foster matching.

### 3 Problem Statement: A Broadband Electrically-Short Monopole for VHF-Low

In light of the above considerations, we now pose the specific problem that will be addressed in Phase III of this project: Using the Phase I 23.5 cm high  $\times$  5 mm radius monopole from [9], further analyzed as the Phase II base element in [10], develop a system derived from the NIC design described in [6] that achieves acceptable performance for the entire VHF-Low band (25–50 MHz); and, secondarily, optimize that performance.

To more clearly illustrate the nature of the problem, we consider the performance of traditional (i.e., *not* non-Foster) solutions to the above problem using data which we originally reported in [10]. We begin with the self-impedance  $Z_A$  of the antenna, shown in Figure 1. As expected, the reactance ( $X_A$ ) is large and negative, ranging from  $-1583 \Omega$  at 25 MHz to  $-776 \Omega$  at 50 MHz. Also note that the real part of the impedance ( $R_A$ ) is quite small relative to both the antenna reactance and the nominal characteristic impedance ( $50 \Omega$ ), ranging from  $0.19 \Omega$  at 25 MHz to  $0.75 \Omega$  at 50 MHz.

There two methods by which we may anticipate the limits of impedance matching in this case. One approach is to consider that the nominal fractional bandwidth  $B$  is approximately equal to the reciprocal of the intrinsic quality factor  $Q$  of the antenna. The quality factor in this case is approximately equal to

$$Q \approx \frac{1}{(kb)^3} + \frac{1}{kb} \quad (1)$$

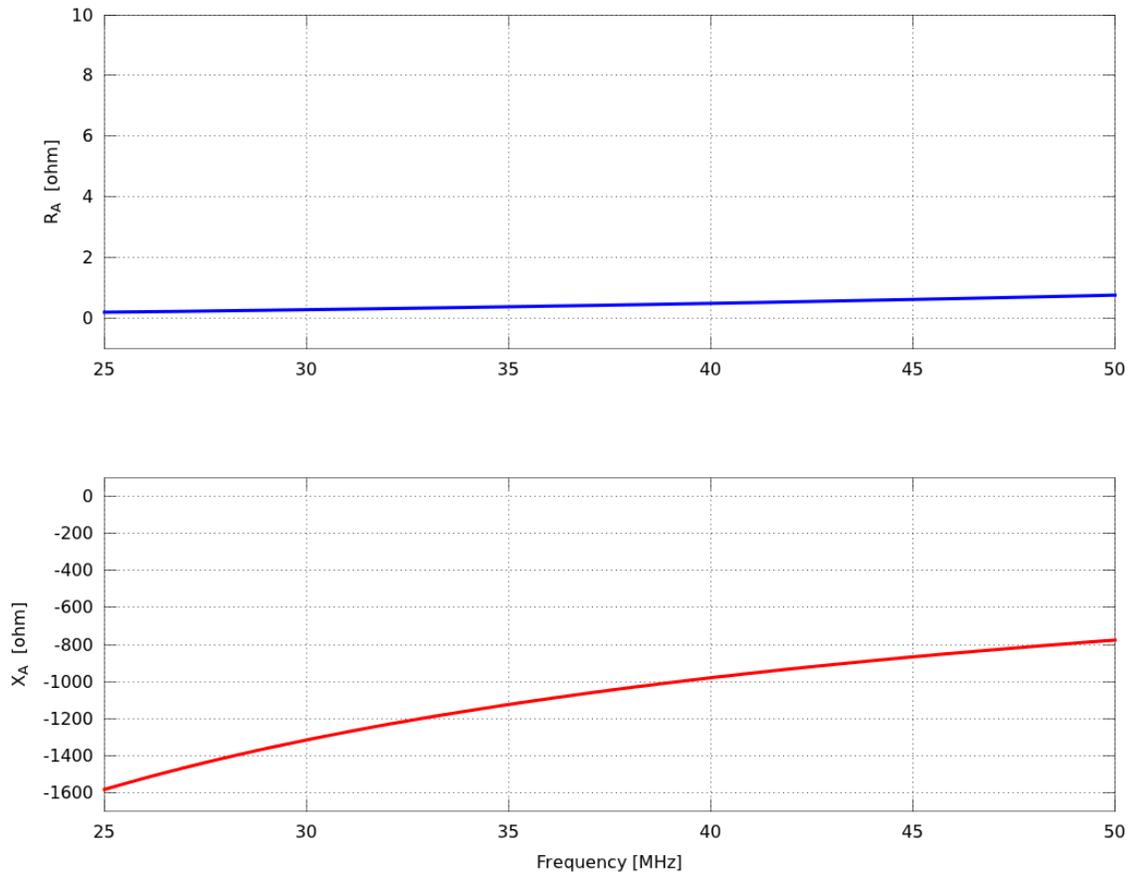


Figure 1: Real ( $R_A$ ) and imaginary ( $X_A$ ) parts of the impedance  $Z_A$  of a 23.5 cm high  $\times$  5 mm radius monopole over an infinite perfectly conducting ground plane.

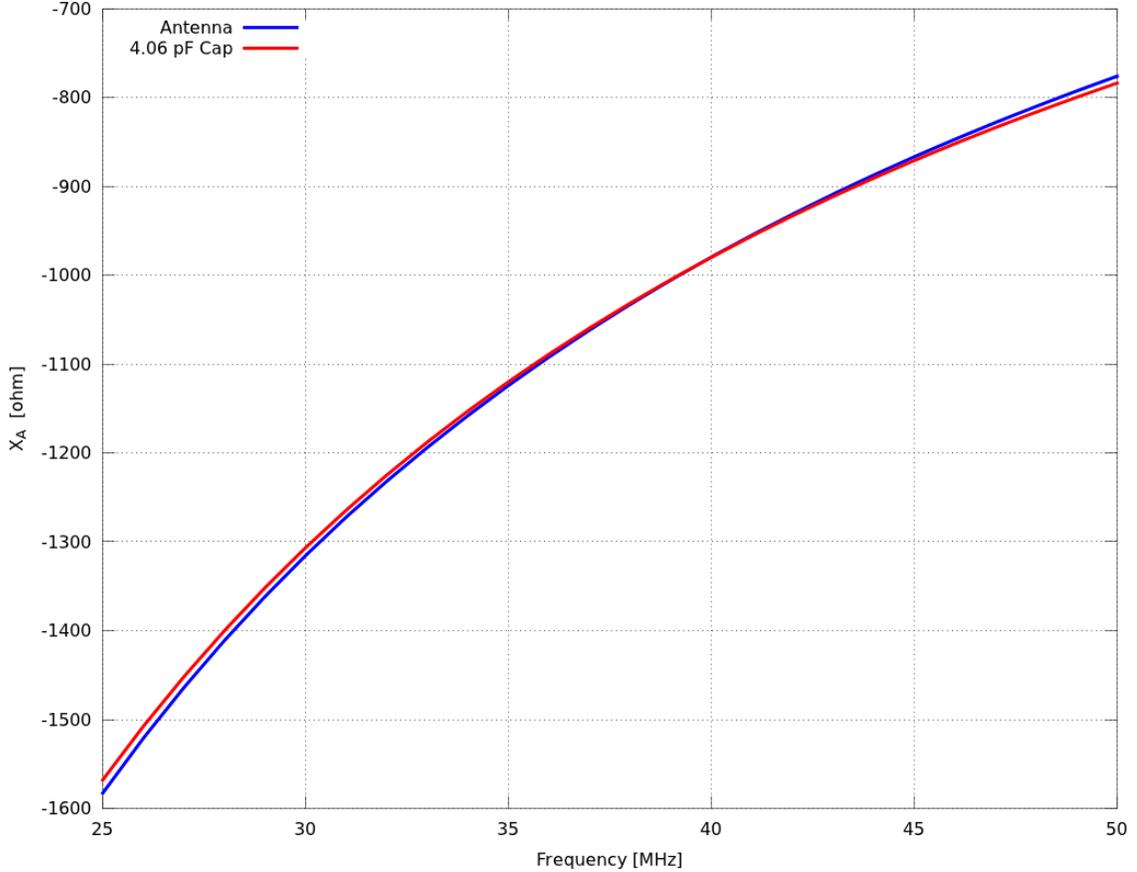


Figure 2: Reactance of the antenna compared to the reactance of a 4.06 pF capacitor.

where  $k = 2\pi/\lambda$ ,  $\lambda$  is wavelength, and  $b$  is the radius of a sphere which just barely encloses the antenna. In this case, we have  $Q \approx 1059$ , so  $B \approx 0.09\%$ , which is just 38 kHz at 40 MHz. An alternative approach to the same question is as follows: Figure 2 demonstrates that the reactance of the antenna in 25–50 MHz is well-modeled as that of a 4.06 pF capacitor. Since the real component of the antenna impedance is orders of magnitude smaller, it is reasonable to model the antenna impedance as a  $0.478 \Omega$  resistor in series with a 4.06 pF capacitor. From Fano-Bode theory, the bandwidth  $B_{max}$  over which  $VSWR \leq 2$  can be obtained for this circuit is approximately upper-bounded by [8]

$$B_{max} \approx \frac{2\pi^2 f_0 RC}{\ln 3} \quad (2)$$

where  $f_0$  is the nominal frequency, and  $R$  and  $C$  are the values of the series resistance and series capacitance, respectively. For  $f_0 = 40$  MHz,  $R = 0.478 \Omega$  and  $C = 4.06$  pF, we obtain  $B_{max} \approx 0.14\%$ , which is just 56 kHz at 40 MHz. Thus we find that the upper bounds on bandwidth for  $VSWR \leq 2$  (i.e., 38 kHz and 56 kHz) are in reasonable agreement given the nature of the approximations, and indicate that using the theoretically best-possible match the bandwidth that could be expected would be no greater than the span occupied by 3–4 adjacent 12.5-kHz LMR channels. Thus any match which could improve the  $VSWR \leq 2$  bandwidth to cover a large fraction of the VHF-Low band would have to be non-Foster in nature, and would be very impressive.

Furthermore, note that the above bounds represent theoretical limits, and that any practical match will be much worse. For example, one might consider a simple match consisting of a series inductor

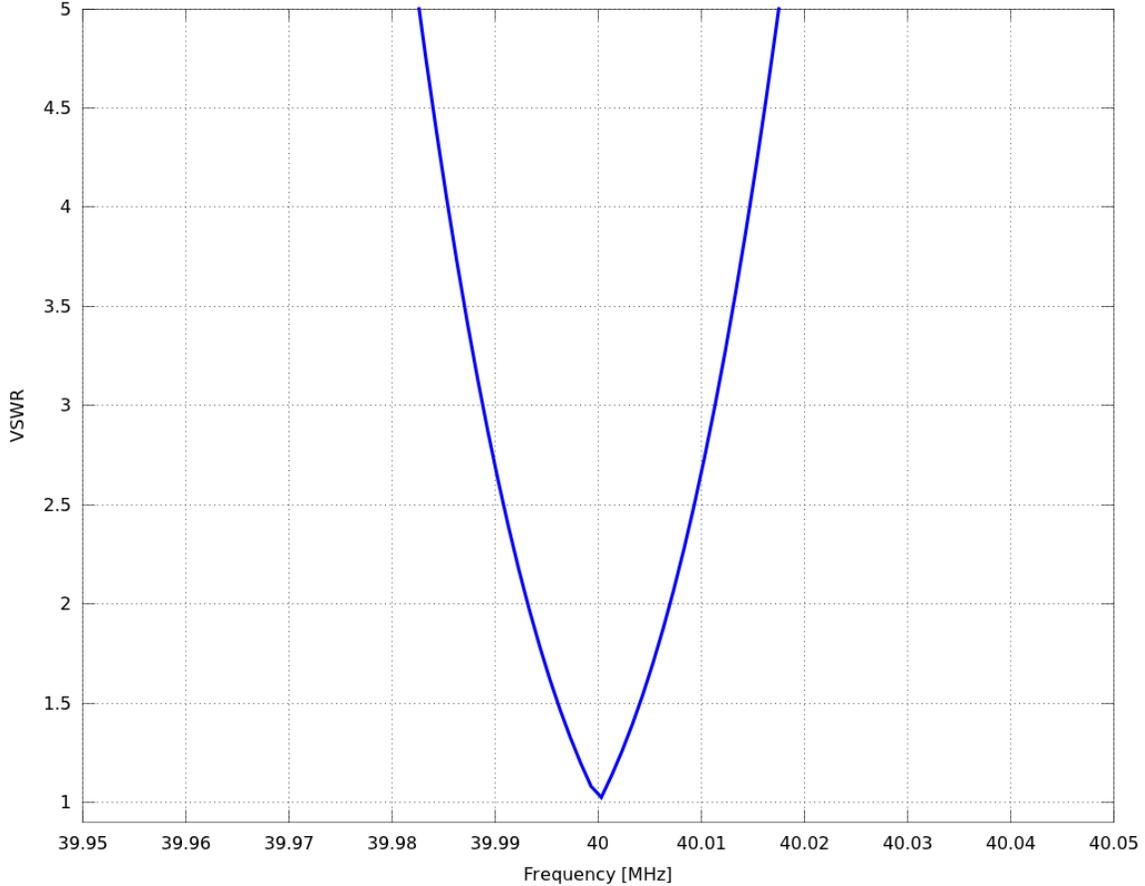


Figure 3: VSWR after matching the 23 cm monopole with  $3.90 \mu\text{H}$  series inductance followed by a transformer to match the real part of the impedance. Note that the frequency span is just 100 kHz around 40 MHz.

to cancel the negative reactance of the antenna at 40 MHz, followed by a transformer to match the real part of the impedance. The required inductor is  $3.90 \mu\text{H}$ . Figure 3 shows the resulting VSWR for this match. Note the bandwidth for  $\text{VSWR} \leq 2$  is about 15 kHz, which is consistent with (i.e., much less than) the bounds estimated above.

## 4 Non-Foster Design Considerations

To begin, let us consider how much the bandwidth could possibly be increased using a series non-Foster element. Figure 4 shows the VSWR obtained using a series matching reactance equal but opposite in sign to that of the antenna, thus zeroing the antenna reactance, followed by a 1:11 transformer to match the real part of the antenna impedance to  $50 \Omega$  at 40 MHz. Note the VSWR is nearly 2 or better over 25–50 MHz. Therefore a series non-Foster reactance alone is, in principle, sufficient to obtain the desired performance.

Next we assume a perfect NIC is available, and we use it to transform a capacitance of  $4.06 \text{ pF}$  (from Figure 2) in the associated negative capacitance with the goal of producing a suitable canceling reactance. Figure 5 shows the result. Although Figure 2 indicates that the antenna reactance is well-modeled as a  $4.06 \text{ pF}$  capacitor over this frequency range, the error is nevertheless large enough to dominate the total impedance. This is because the real part of the impedance is relatively small

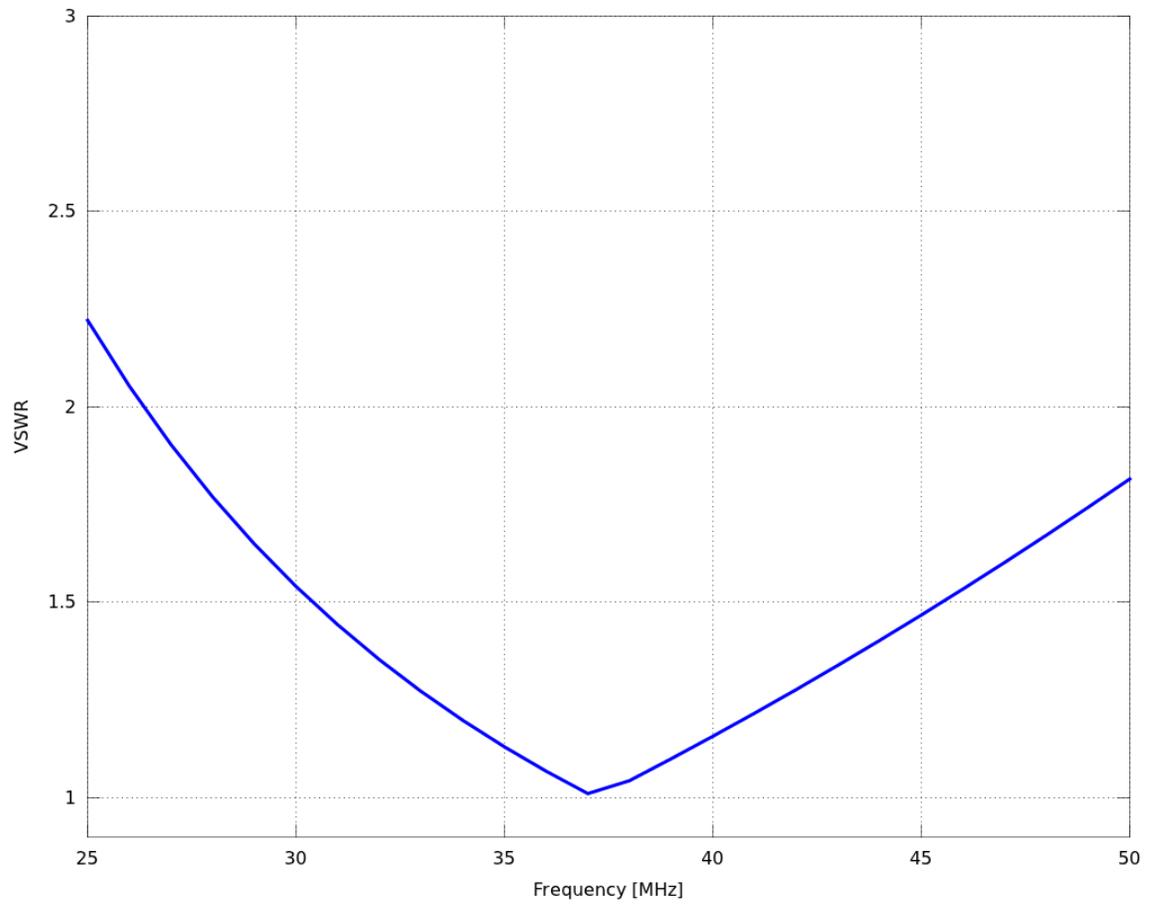


Figure 4: VSWR after zeroing the reactance of the 23 cm monopole using a series non-Foster reactance, followed by a 1:11 transformer to match the real part of the impedance.

– only about  $0.5 \Omega$  at 40 MHz. As a result, the VSWR bandwidth is not much better than can be achieved using conventional passive matching. To do better, a reactance which more precisely matches the antenna reactance is required.

After some trial and error, a series combination of a 4 pF capacitance and a 60 nH inductance was found to yield sufficiently good results, shown in Figure 6. The frequency range for  $VSWR \leq 2$  is now about 37–45 MHz ( $\approx 20\%$  bandwidth), which is dramatically better than the bandwidth that can be achieved by passive matching ( $< 0.1\%$ , as demonstrated earlier). No doubt this could be further improved by additional fine adjustments to the capacitance and inductance; however, this is probably not reasonable in practice since the both the actual reactance of the antenna could be significantly different, and it is quite difficult to precisely set capacitance and inductance at the low levels needed here.

## 5 A Preliminary System Design

Based on the above considerations, Figure 7 shows a preliminary design for a 23 cm high  $\times$  5 mm radius monopole that could be usable over the entire 25–50 MHz range. The NIC is the design from [6]. The capacitance is variable over 1–10 pF to account for the likelihood that *in situ* tuning will be necessary due to the inability to precisely estimate the antenna reactance in advance, and also due to limits in the precision with which physical reactances can be implemented. The variable capacitor would most likely be implemented as a varactor diode (also known as “varicap”).

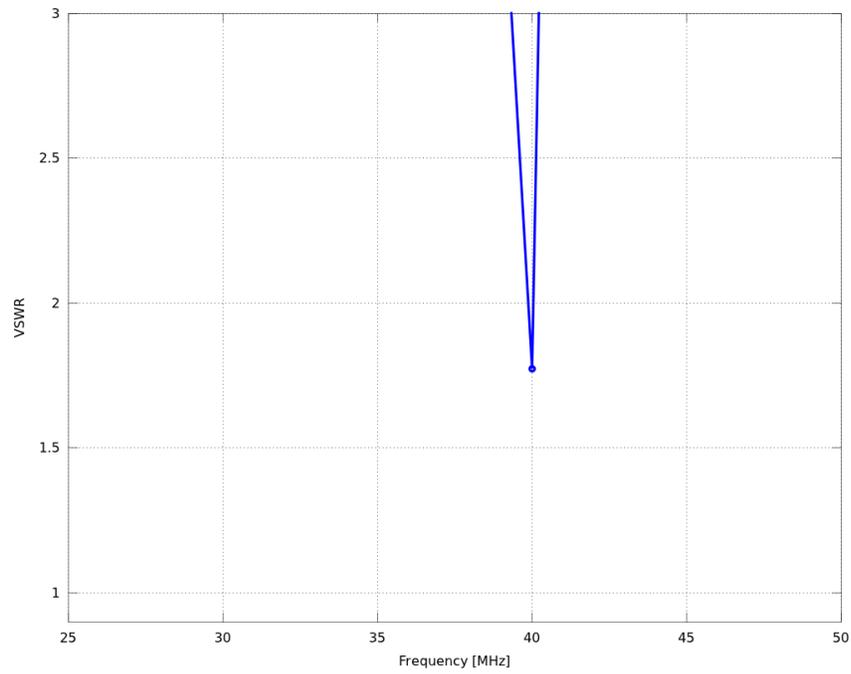
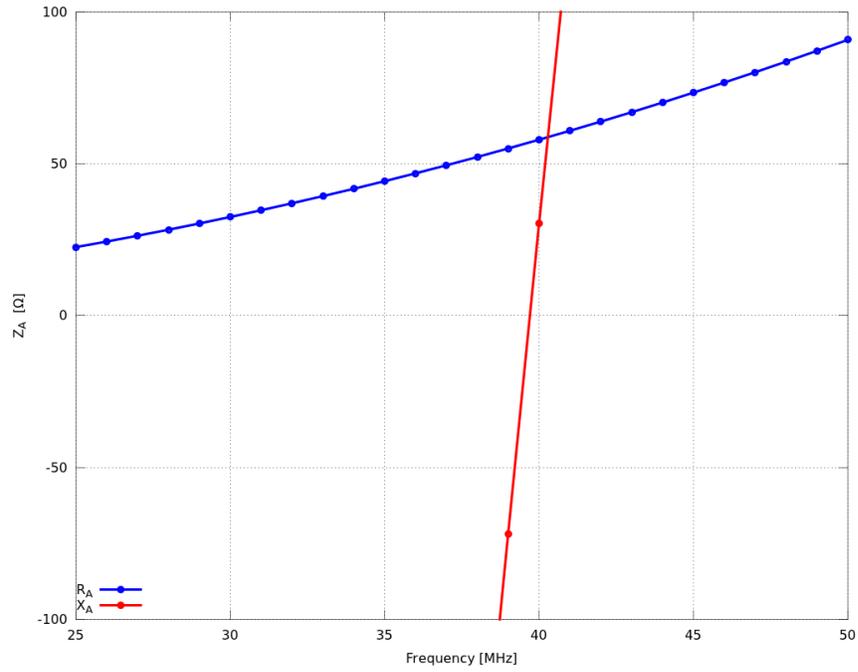


Figure 5: Matching a 23 cm monopole using a  $-4.06$  pF series capacitor followed by a 1:11 transformer to match the real part of the impedance. *top*: Impedance looking into transformer from transceiver. *bottom*: VSWR with respect to  $50 \Omega$ .

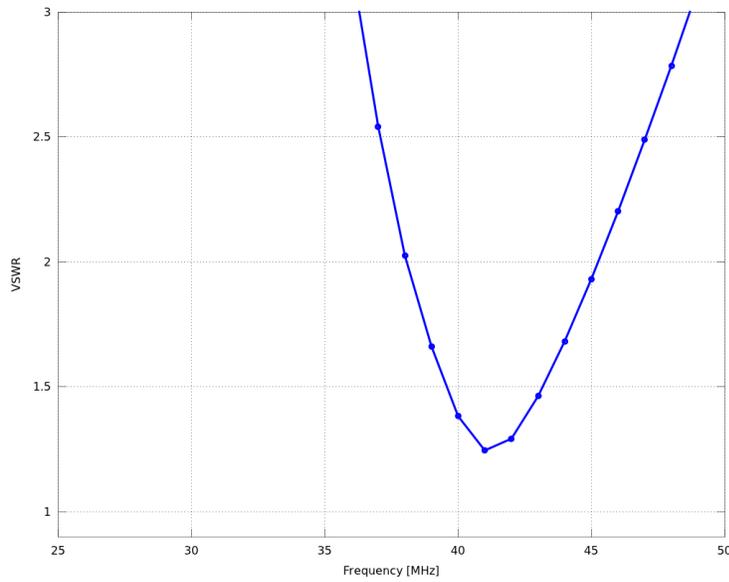
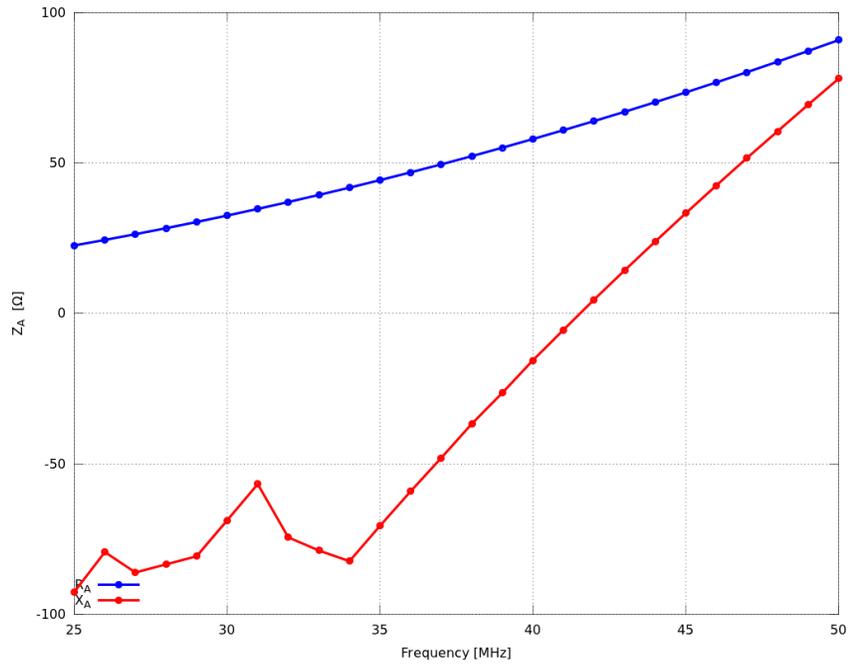


Figure 6: Matching a 23 cm monopole using a series combination of a  $-4$  pF series capacitance and  $-60$  nH inductance, followed by a 1:11 transformer to match the real part of the impedance. *top*: Impedance looking into transformer from transceiver. *bottom*: VSWR with respect to  $50 \Omega$ .

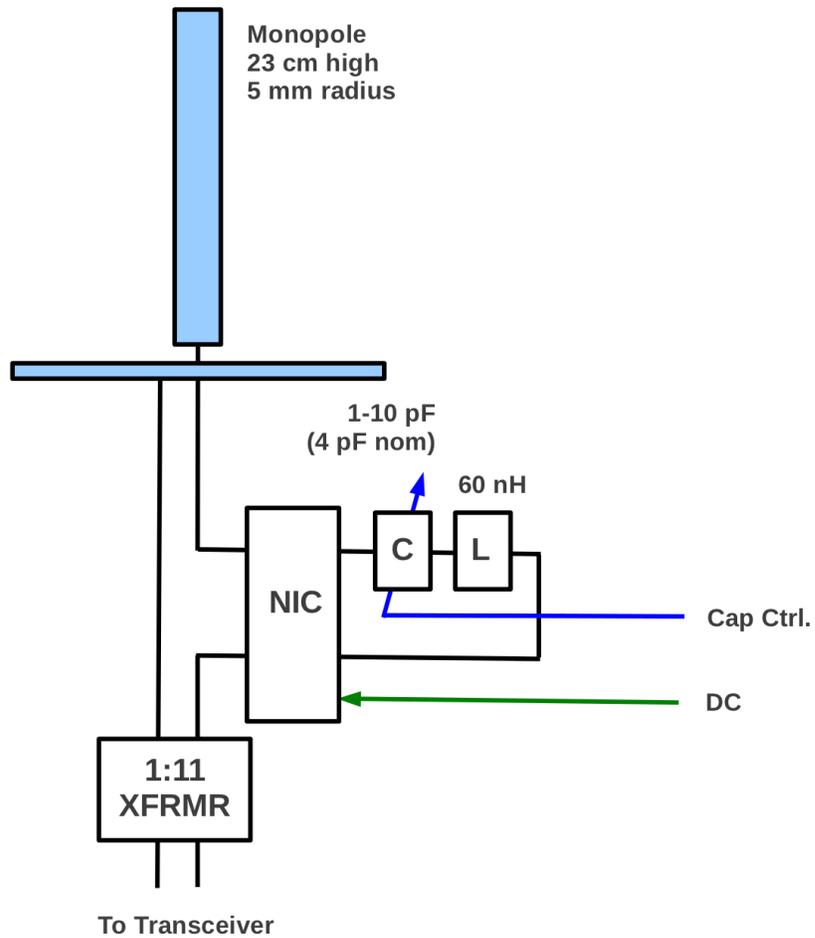


Figure 7: Preliminary design for a non-reconfigurable 23-cm monopole for the VHF-Low band using non-Foster matching.

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