Optimization of the Bandwidth of Electrically Small Planar Antennas

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Abstract: The bandwidth limitations of resonant, electrically small antennas that fully occupy a spherical volume are well defined by the inverse of the Chu-limit or the lower bound on quality factor. Recently, Gustafsson et al developed lower bounds for the *Q* of antennas of arbitrary shape. We have previously described antenna designs fully occupying a spherical volume and a cylindrical volume that exhibit *Q's* close to the Chu-limit and close to the Gustafsson limit, respectively.

In many applications, there is a requirement for electrically small planar antennas. It is understood that the *Q* of an electrically small planar antenna cannot approach the *Q* of a spherical or cylindrical antenna, where all have the same value of *ka* and fully occupy their respective total available volumes. Comparing the *Q's* of planar antennas to the lower bound for spherical or cylindrical antennas does not provide sufficient insight into how well the antenna performs relative to the theoretical performance that can be achieved in the planar shape. Recently, Gustafsson et al presented the limit on achievable *Q* for planar antennas as a function of their length-to-diameter ratio. In this paper, we present several designs for electrically small, thin planar antennas and compare their *Q's* to the Gustafsson limit.

Introduction

When designing electrically small antennas there are a number of performance properties to consider. These include the antenna's impedance, which provides information on the match to the transmitter and/or receiver as well as the antenna's operating bandwidth; the antenna's radiation efficiency; and the antenna's radiation patterns.

Most electrically small antennas exhibit radiation patterns consistent with those of a fundamental, single mode dipole or monopole. In some instances, where the radiating structure is arbitrarily shaped or includes an arbitrarily shaped ground plane, the radiation pattern may deviate from that of a fundamental dipole or monopole. In many wireless applications, where the link is principally established through multipath, the antenna's radiation pattern may be a secondary concern.

The small antenna's radiation efficiency can often be made reasonably high with suitable design approaches and suitable choice of materials. However, there are instances where a low radiation efficiency may be necessary or desirable in order to increase the operating bandwidth of the antenna to meet system requirements and/or make it less sensitive to frequency shifts that may occur due to changes in the operating environment.

Impedance matching the electrically small antenna at a single frequency is often straight-forward through changes in the antenna design or easy using external lumped matching components. When comparing the performance properties of different electrically small antenna, presumably where they have the same size and occupy the same volume, the critical performance property to consider is bandwidth or *Q*. These are generally the limiting performance properties in the design of an electrically small antenna of a given size.

In this paper, we focus on the design of several planar antennas with the purpose of comparing their *Q's* to the lower bound defined by Gustafsson et al. We verify that, as predicted by Gustafsson, there is an optimum value of length-to-diameter ratio for achieving the lowest *Q* or the widest possible bandwidth. We assume here that all of the antennas exhibit a single resonance within their impedance bandwidth and exhibit fundamental mode radiation patterns.

Background

The material that appears in this section has been previously published in the open literature[1] and is included here for completeness.

The lower bound on *Q* [1]-[2], Q_{lb}, often referred to as the Chu-limit, establishes the theoretical minimum value of *Q* that can be achieved as a function of the antenna's occupied spherical volume, which is defined by the

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[1] The text contained in this section has been extracted and edited from the following: S. R. Best, "A Comparison of the Cylindrical Folded Helix *Q* to the Gustafsson Limit," *EuCap 2009*, Berlin, Germany, March 2009.

value of *ka*, where *k* is the free space wavenumber $2\pi/\lambda$, and *a* is the radius of an imaginary sphere circumscribing the maximum dimension of the antenna. The lower bound on *Q* for the general, single mode (fundamental TE or TM mode), lossy antenna is given by

$$Q_{lb} = \eta_r \left(\frac{1}{(ka)^3} + \frac{1}{ka} \right) \quad (1)$$

where η_r is the antenna's radiation efficiency.

To achieve a *Q* that most closely approaches the lower bound of Equation 1, the small antenna must utilize the full spherical volume defined by the value of *ka*. The lowest possible *Q* is achieved when the antenna conductor(s) are confined to the outermost regions of the spherical volume [3]−[5].

In most practical applications, the constraint on the occupied volume of a small antenna is not defined by a spherical shape. Typically, the small antenna must fit within a volume of arbitrary shape or in many cases, a cylindrical or planar shape. In these instances, the antenna *Q* will not approach the lower bound as closely as does the *Q* of the spherical shaped antenna. Without an appropriate adjustment in the lower bound of Equation 1 for differences in antenna shape, the engineer has no measure of how well the arbitrary shaped antenna performances relative to theoretical limits.

Recently, Gustafsson et al derived a lower bound on *Q* for arbitrary shaped antennas [6], thus providing the engineer with the capability of determining how well the general small antenna performs relative to theoretical limits. Gustafsson also defined specific lower bounds for antennas having a cylindrical shape [6] - [7], and recently, antennas having a planar shape.

The exact *Q* of an electrically small, tuned or self-resonant antenna is given by [1], [8]

$$Q(\omega_0) = \frac{\omega_0 |W|}{P} \quad (2)$$

where *W* is internal energy and *P* is the total power accepted by the antenna, which includes both power dissipated in the form of radiation and heat within the antenna structure. ω_0 is the radian frequency ($2\pi f_0$) where the antenna is naturally self-resonant, tuned, or made to be self-resonant. If the tuned small antenna exhibits a single impedance resonance within its defined *VSWR* bandwidth, its *Q* can be accurately approximated at any frequency, ω, from its impedance properties using [8]

$$Q(\omega) \approx \frac{\omega}{2R(\omega)} \sqrt{R'(\omega)^2 + \left(X'(\omega) + \frac{|X(\omega)|}{\omega} \right)^2} \qquad (3)$$

where $R'(\omega)$ and $X'(\omega)$ are the frequency derivatives of the antenna's feed point resistance and reactance, respectively.

In recent years there has been substantial interest in developing electrically small antennas that are impedance matched, exhibit low *Q* and have high radiation efficiency. One antenna that was designed with the objective of achieving these characteristics is the folded spherical helix [3] – [4] depicted in Figure 1.

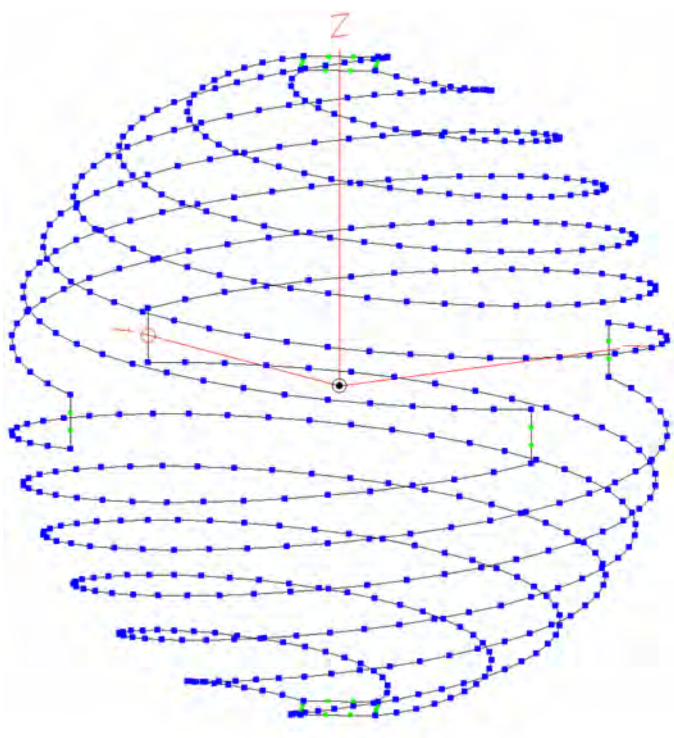


Fig. 1 The 4-arm spherical folded helix antenna.

The advantage of the folded spherical helix design is that it utilizes the entire spherical volume where all of the conductors are wound on the outside of the imaginary spherical shape. Multiple folded arms are used to both impedance match the antenna and reduce its *Q*. In this case, four folded arms are used to match the antenna to a 50Ω characteristic impedance. There is a single feed point in the antenna at the center of one of the short vertical sections of conductor.

At a value of *ka* = 0.265, the folded spherical helix is self-resonant with a total resistance (including both radiation and loss terms) of 47.6Ω, a radiation efficiency of 97.1% and a *Q* of 84.64, approximately 1.52 times the lower bound of 55.61. For *ka* < 0.5, this value of *Q* is consistent with the practical, minimum achievable *Q* predicted by Thal for spherical wire antennas [5].

The spherical folded helix discussed above utilizes the full spherical volume defined by a specific value of *ka*. In doing so, the design is able to be adjusted to achieve a *Q* that closely approaches the lower bound. Similarly, for the cylindrical shape, a cylindrical folded helix was designed using the same principles with the exception that the conductors are wound on the outside surface of an imaginary cylinder. The dimensions of the antenna (its overall length, overall diameter and conductor length) are set so as to maintain self-resonance at the same value of *ka* (0.265) as that of the folded spherical helix.

In a design approach similar to that used with the folded spherical helix, self-resonance is achieved by adjusting the arm length in each of the folded arms. Adjustment of the arm length changes the total self-inductance of the structure, tuning out the inherent self-capacitance associated with the small dipole-like design. Once self-resonance is achieved, the resonant resistance is increased by adding folded arms to the structure. Since the cylindrical folded helix does not occupy the same overall volume as the spherical folded helix having the same value of *ka*, it will not exhibit as low a *Q*. The basic configuration of a 4-arm cylindrical folded helix is depicted in Figure 2.

In encompassing the cylindrical folded helix within the same spherical volume (*ka*) as the folded spherical helix, there are a vast number of length-to-diameter ratios that can be chosen. As expected and quantified by Gustafsson et al, the minimum achievable *Q* for the antenna will vary as a function of its length-to-diameter ratio. Additionally, with this type of antenna design, the resonant resistance is a function of $(\ell/\lambda)^2$, where ℓ is the overall length of the cylinder. As a result, the resonant radiation resistance and antenna *VSWR* will also change as a function of length-to-diameter ratio for a fixed number of turns.

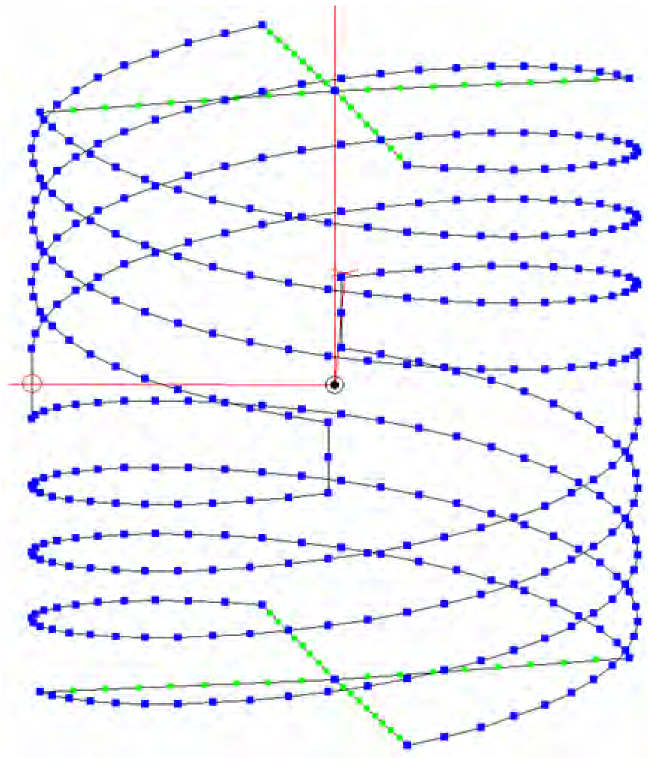


Fig. 2 The 4-arm cylindrical folded helix antenna.

A number of 4-arm folded helices with different length to diameter ratios were studied. The ratio of the antenna *Q* to the lower bound (the Chu limit) of Equation 1 was calculated and compared against the Gustafsson limit for cylindrical shaped antennas [7]. This comparison is presented in Figure 3. Note that in calculating the Gustafsson limit presented in Figure 3, it was assumed that the antennas have a *ka* much less than 1, are purely vertically polarized and that the maximum achievable directivity is 1.5. In all cases, the *Q* of the antenna is above or at the Gustafsson limit.

The objective of the present work is to perform a similar study for planar antennas as a function of their length-to-diameter ratio. We compare a number of different planar antennas, consider simple techniques to impedance match them and then compare their *Q's* to the Gustafsson limit.

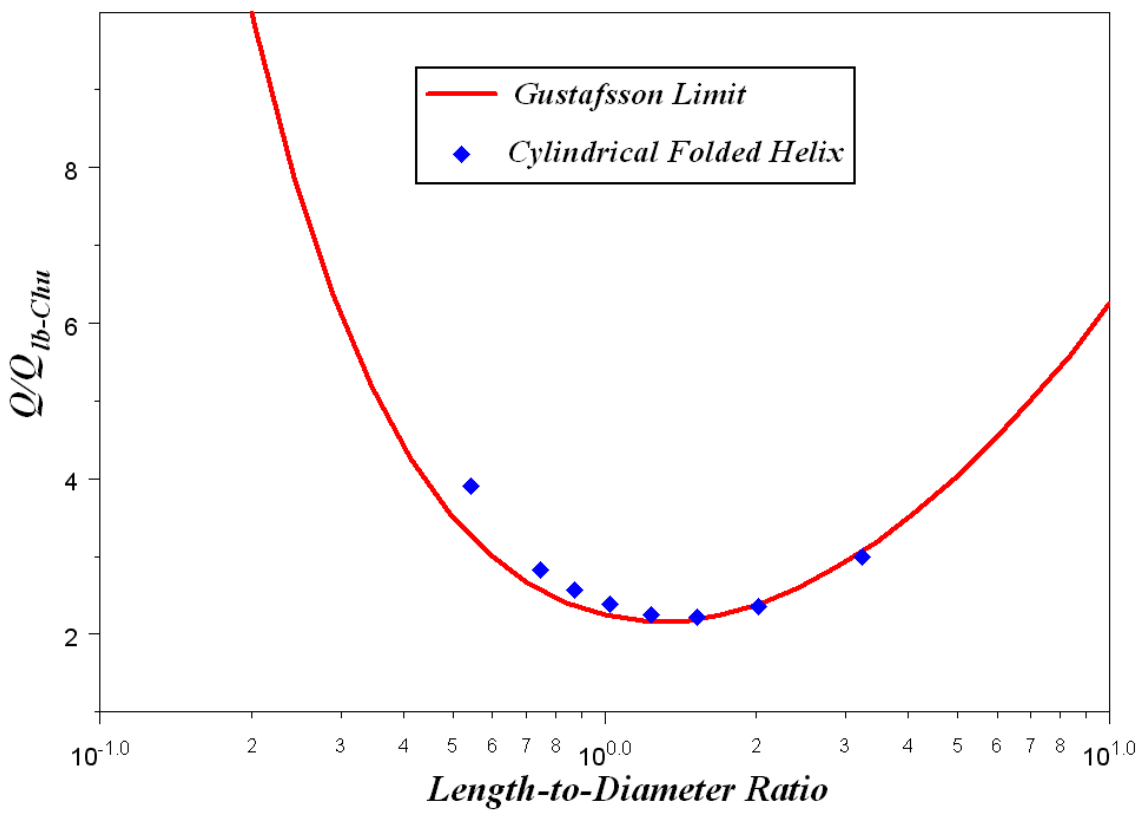


Fig. 3 Comparison of the cylindrical folded helix Q to the Gustafsson limit.

Planar Antennas

In the previous section, we described two similar antenna designs that exhibit *Q's* which approach and meet theoretical limits. In this section, we describe several planar antennas that are designed with the same objective, namely to develop antennas having a planar geometry which exhibit a matched impedance and *Q's* that meet theoretical limits. While the Chu-limit provides an absolute minimum for antenna Q as a function of occupied spherical volume, it provides no insight into what value of Q can be practically achieved using a planar geometry.

Gustafsson provided data for the minimum achievable Q for planar antennas as a function of their length-to-diameter ratio [7]. It was assumed that the antennas are electrically small, exhibit a single resonance within their defined operating bandwidth and that they have a directivity equal to 1.5. A plot of the Gustafsson limit, expressed as the ratio of antenna Q to the Chu-Limit (Q/Q_{lb}), is presented in Figure 4. From Figure 4, it is seen that the optimum length-to-diameter ratio for the planar dipole antenna is approximately 1.9.

The design approach used with the planar antennas describe here is essentially identical to that used with the spherical and cylindrical folded helices with the exception that the antenna geometry is confined to a 2D planar area. The antenna designs presented here are straight-forward. Their initial dimensions were chosen to approximate the length of the two helices, having a nominal dipole length of approximately 8.36 cm. The width of the planar antennas was varied to change the antenna length-to-diameter ratio. With only a few exceptions, all of the antennas considered here have a conductor diameter of 1 mm and they were designed to operate near a frequency of 300 MHz, where the value of *ka* would be less than 0.5.

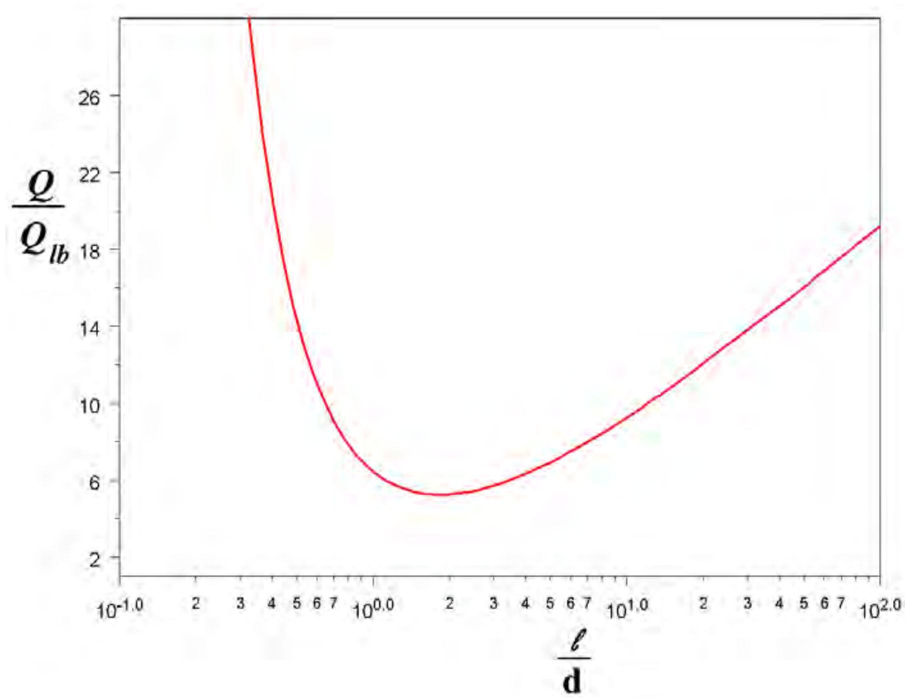


Figure 4. The Gustafsson limit for electrically small planar antennas as a function of length-to-diameter ratio.

The initial and perhaps most obvious geometry for designing an electrically small planar antenna is a meander line structure as illustrated by the meander line configurations presented in Figure 5. The first antenna considered is meander line M1, which has an overall dipole length of 7.96 cm and a diameter of 4 cm, corresponding to a length-to-diameter ratio of 1.99. It is resonant at a frequency of 329.7 MHz, corresponding to a *ka* equal to

0.308. It has a resonant radiation resistance of 3.1Ω and a Q of 166.8[2], corresponding to a Q/Q_{lb} ratio of 4.4. Its radiation efficiency is 63.7%. Before comparing its Q to the Gustafsson limit, we will describe a number of other small planar designs considered here.

In many applications, it is desirable to design the antenna to exhibit a resonant impedance equal to 50Ω. In some applications, such as small super directive arrays, it may be desirable that the antenna exhibit a substantially higher resonant resistance. The other meander line antennas presented in Figure 5 are designs implemented to achieve an impedance match relative to 50Ω, or higher impedance as may be necessary in other applications. Achieving an impedance match with the small planar antennas considered in this work is somewhat trivial as it is easily implemented using a near-field reactive coupling matching element approach or a shunt-stub, parallel inductor approach.

The other configurations presented in Figure 5 are design variations that match the antenna impedance to 50Ω or another, higher arbitrary value. Configurations M2 through M4 use a near-field, reactive coupling matching approach where the electrically small dipole, to the left of the meander line, is matched using the resonant meander line structure. This approach is a common technique used in impedance matching electrically small loops and it has more recently been used by Erentok and Ziolkowski in many of their small antenna designs [9]. Configuration M5 uses a shunt-stub (parallel inductor) to match the meander line antenna to 50Ω. A comparison of the antenna impedances is presented in Figure 6. As is seen in Figure 6, the antennas can be easily modified for an impedance match to 50Ω or another, higher characteristic impedance. The antennas' other performance properties will be compared shortly after discussing several other planar antenna designs.

In addition to the simple meander line antennas shown in Figure 5, there are numerous antenna configurations and geometries that may be used to implement electrically small planar antennas. Only a few representative configurations can be described here. Several alternate approaches to designing small planar antennas are illustrated in Figures 7 and 8.

The antennas presented in Figure 7 are meander line geometries where the meandering is oriented in a manner similar to that of a planar inductor. This approach to winding the conductor is generally more effective in that it

[2] In this paper, we calculate the Q's of the antennas assuming that the wires are perfectly conducting.

requires less total wire length to achieve resonance relative to the configurations presented in Figure 5. One of the performance drawbacks is that it typically introduces some level of orthogonal TE mode radiation, resulting in minor overhead null fill.

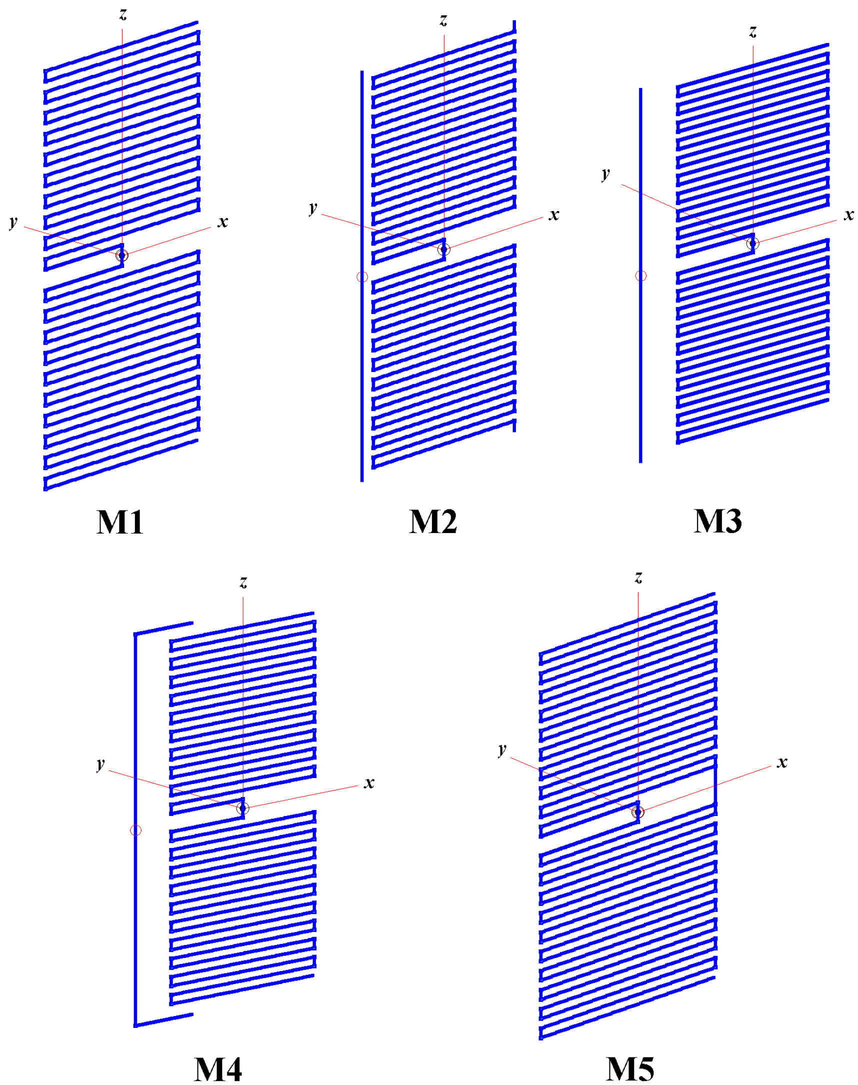


Figure 5. The electrically small planar meander line antennas.

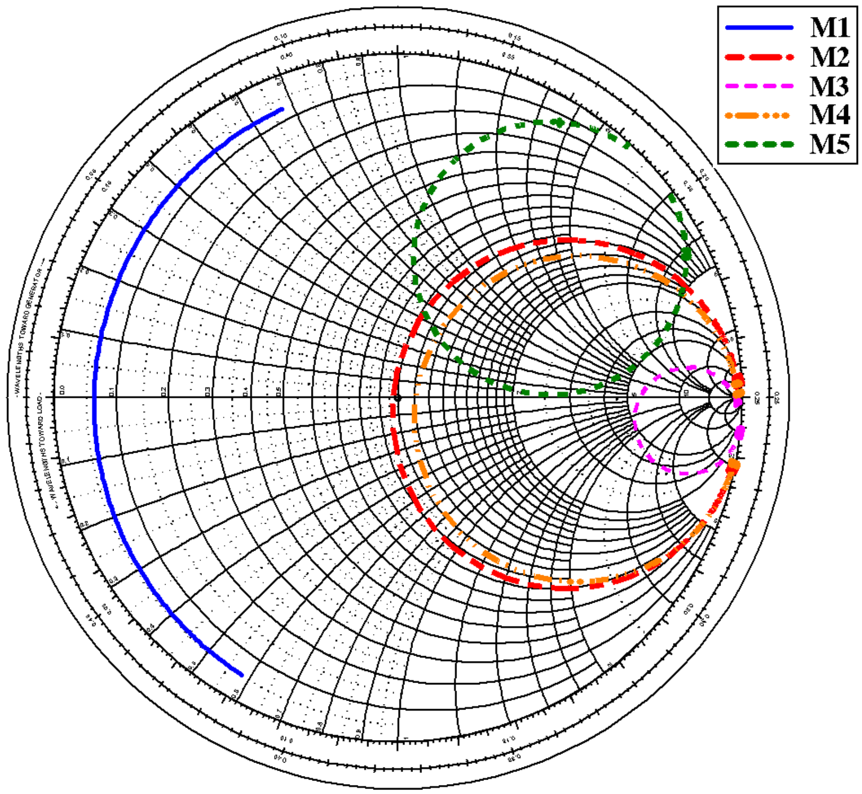


Figure 6. The feed point impedance of the electrically small planar meander line antennas.

Antenna configuration W1 has an overall dipole length of 8.36 cm and a diameter of 4 cm, corresponding to a length-to-diameter ratio of 2.09. It is resonant at a frequency of 319.3 MHz, corresponding to a *ka* equal to 0.31. It has a resonant radiation resistance of 2.3Ω and a *Q* of 325.9, corresponding to a Q/Q_{lb} ratio of 8.86, which is substantially higher than that of meander line M1. Its radiation efficiency is 77.1%.

Antenna configurations W2 and W3 use near field, reactive coupling matching to achieve the impedance match relative to 50Ω and an arbitrary characteristic impedance of 355Ω, respectively. Antenna configuration W4 uses a shunt-stub, parallel inductor approach to achieve an impedance match relative to 50Ω.

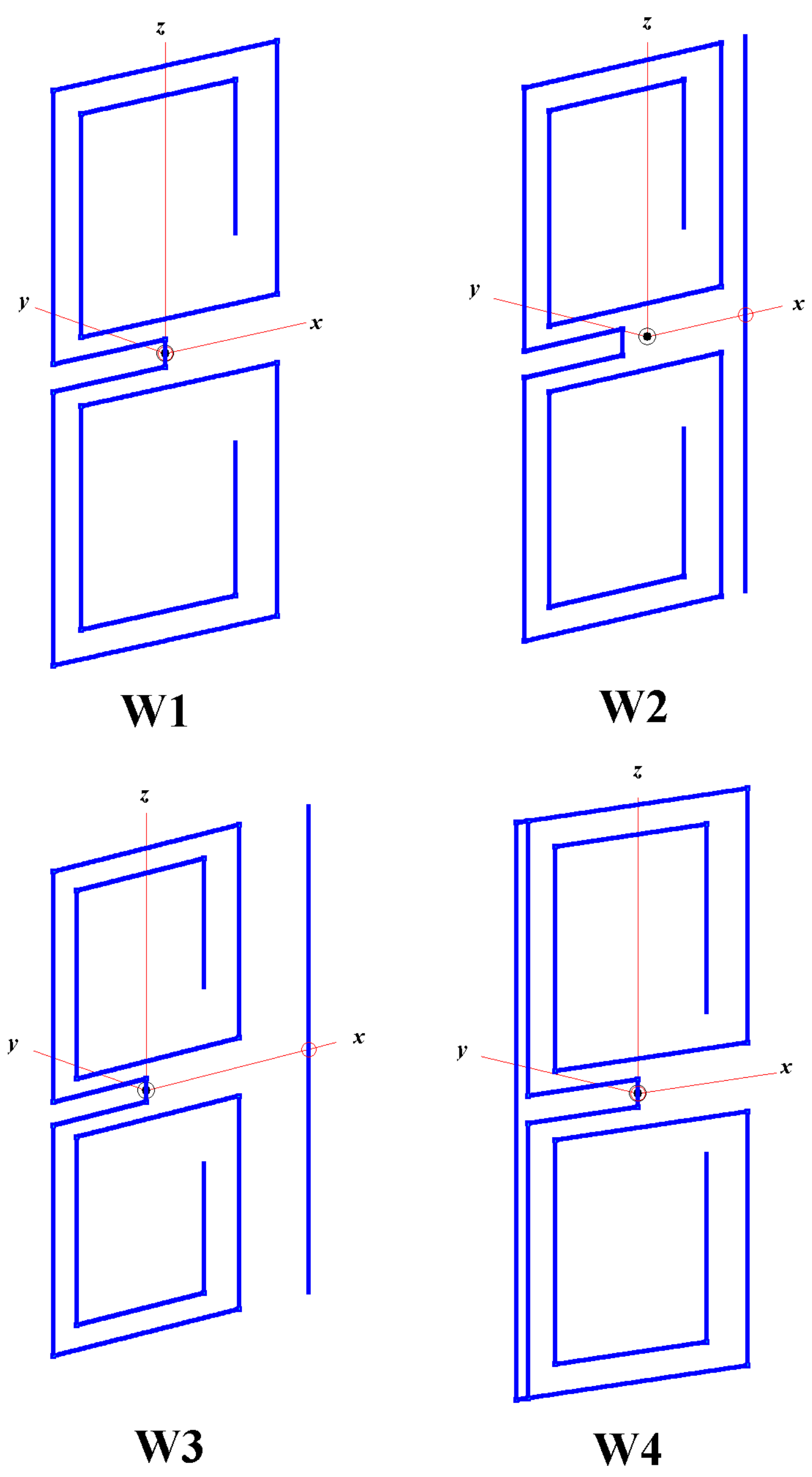


Figure 7. Electrically small planar meander line antennas wound in a manner similar to planar inductors.

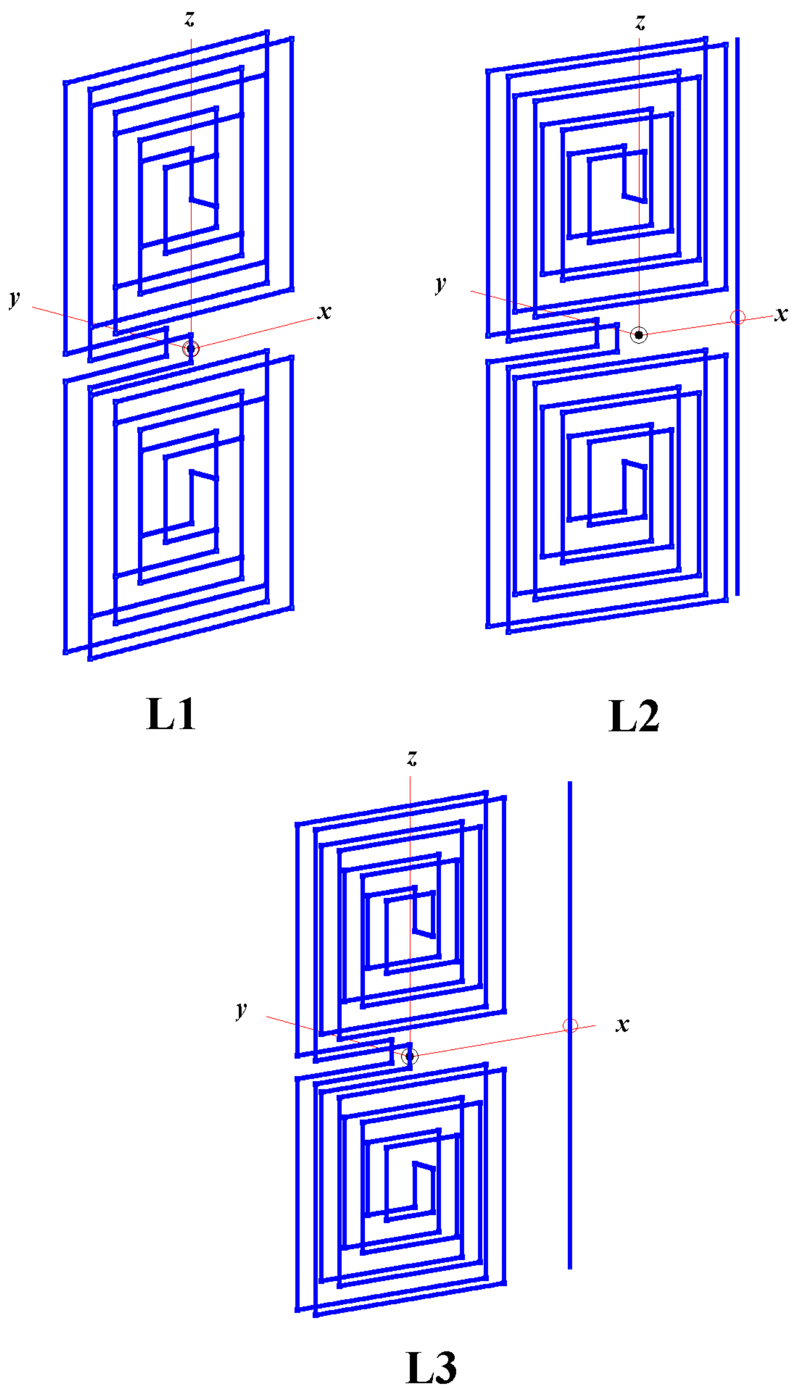


Figure 8. Electrically small planar meander line loop antennas.

The antenna configurations illustrated in Figure 8 are loop antennas derived from the planar meander line antenna W1. The loop nature of the design is created by mirroring the meander line dipole and short-circuiting the two dipoles together at their end points to form a single loop. The increase in wire length in the structure is necessary for a loop antenna to be operated near its first series resonance in nearly the same frequency range as the meander line dipole antennas. These antennas are presented primarily to illustrate that these electrically small planar loops, operated at their first series resonance, will behave in a manner consistent with the small dipole in terms of their performance properties.

Antenna configuration L1 has an overall dipole length of 8.36 cm and a diameter of 4 cm, corresponding to a length-to-diameter ratio of 2.09. It is resonant at a frequency of 245.7 MHz, corresponding to a ka equal to 0.239. It has a resonant radiation resistance of 5.5Ω and a Q of 723.6, corresponding to a Q/Q_{lb} ratio of 9.3, which is higher than that of both meander line M1 and W1. Its radiation efficiency is 70.3%.

Although configuration L1 is a loop antenna in that it is short-circuited at its end points and exhibits a first natural resonance that is an antiresonance, its current distribution and radiation properties are more consistent with those of a small electric dipole rather than a small magnetic dipole. However, much like antenna configuration W1, it does exhibit significant cross polarization and overhead null fill as illustrated in Figure 9. The radiation pattern of meander lines M1 through M5 are identically consistent with those of a straight-wire dipole antenna.

Antenna configurations L2 and L3 use near field, reactive coupling matching to achieve the impedance match relative to 50Ω and an arbitrary characteristic impedance of 304Ω, respectively.

A comparison of the performance properties of all the antennas is presented in Table 1. The ratio of Q/Q_{lb} for each antenna is presented and compared to the Gustafsson limit is Figure 10. We note that all of the antennas can be easily matched to 50Ω using reactive coupling matching or a shunt stub acting as a parallel inductor. The radiation efficiencies of the antennas are reasonable and in some cases can be improved by using a larger conductor diameter. In some cases, such as the M1 through M5 configurations, the conductor diameter cannot be increased due to space limitations. Finally, we note that in terms of antenna quality factor compared to the lower bound, the meander line M1 and M5 offer optimum performance and in fact exhibit a Q/Q_{lb} ratio less than the Gustafsson limit.

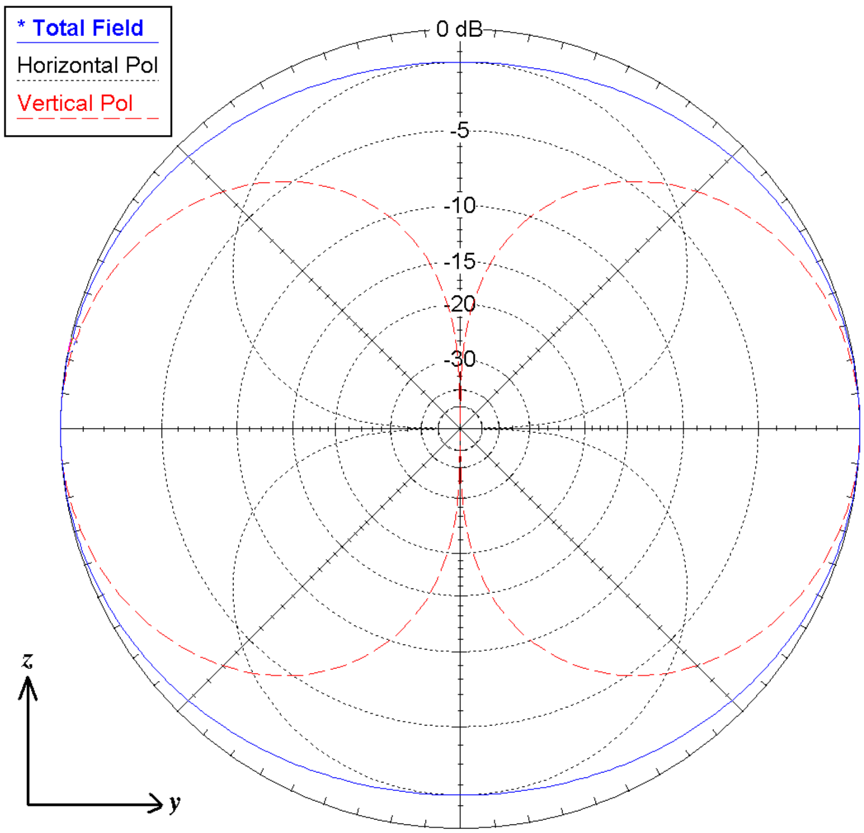


Figure 9. Radiation pattern of the electrically small planar antenna L1. The radiation pattern for the theta sweep plane is shown.

Table 1. Performance comparison of the electrically small planar antennas.

Antenna Configuration	Frequency (MHz)	Overall Height (λ)	*ka*	*l/d*	Radiation Resistance (Ω)	Radiation Efficiency (%)	*Q*	Q/Q_{lb}
M1	329.7	0.088	0.308	1.99	3.1	63.7	166.8	4.44
M2	309.7	0.086	0.305	1.935	48.4	52.0	263.3	6.86
M3	324.8	0.091	0.332	1.672	285.8	59.6	202.4	6.65
M4	315.0	0.090	0.328	1.720	55.4	51.2	279.4	8.93
M5	313.9	0.083	0.301	1.769	65.3	61.8	176.4	4.41
W1	319.3	0.089	0.31	2.09	2.3	77.1	325.9	8.86
W2	309.9	0.087	0.309	1.867	52.5	72.8	418.5	11.32
W3	317.3	0.089	0.334	1.527	355.5	75.7	366.5	12.27
W4	331.5	0.092	0.325	1.99	57.1	79.3	324.6	10.08
L1	245.7	0.069	0.239	2.09	5.5	70.3	723.6	9.3
L2	239.5	0.067	0.237	1.90	35.6	67.14	844.7	10.67
L3	244.6	0.068	0.251	1.64	304.7	69.2	788.9	11.73

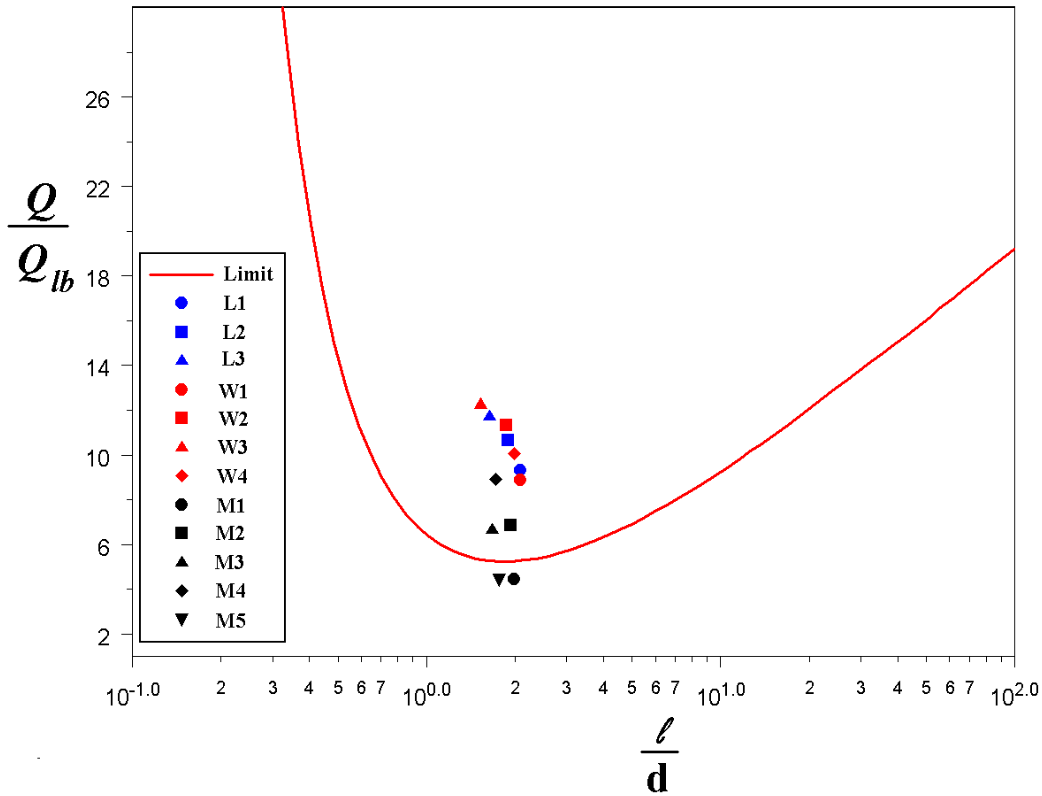


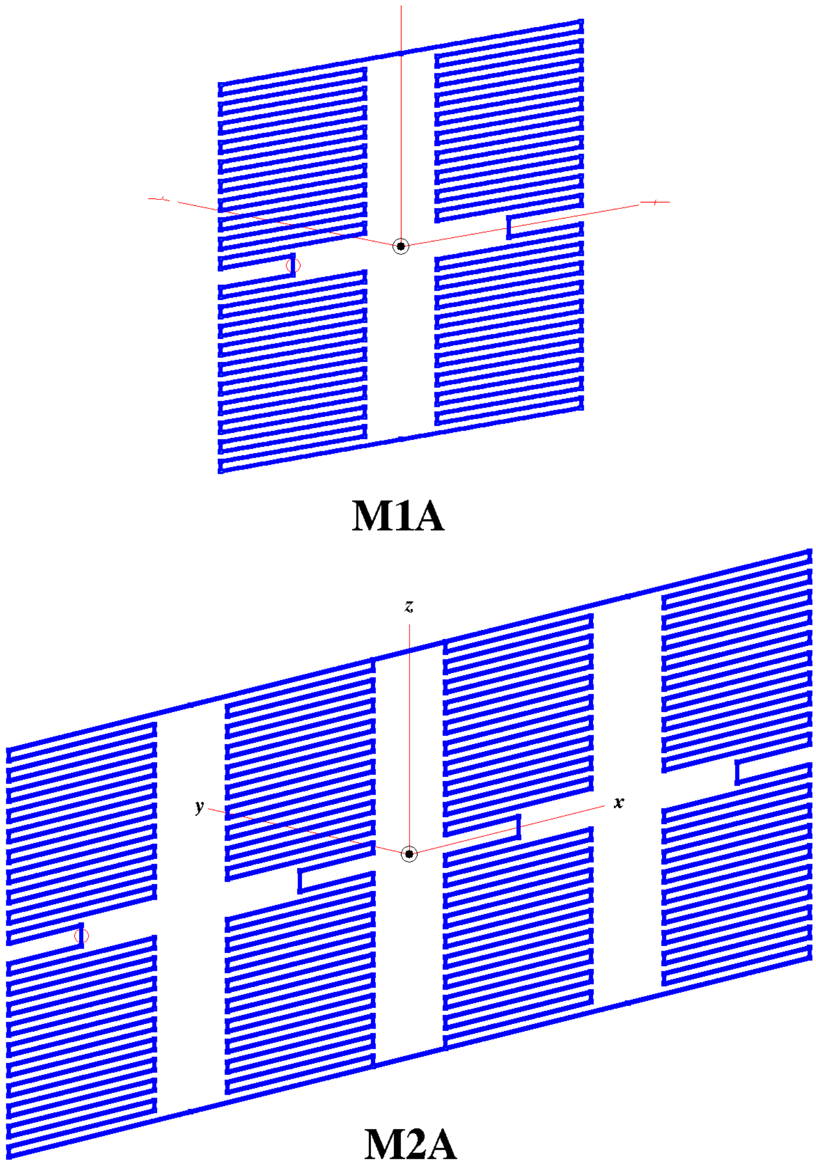
Figure 10. Comparison of the ratio Q/Q_{lb} for the electrically small planar antennas relative to the Gustafsson limit.

The fact that meander line M1 and M5 exhibit a Q/Q_{lb} ratio less than the Gustafsson limit may be expected to some extent as Gustafsson assumes that the lower bound on Q is given by $1/(ka)^3$ rather than Equation 1. To investigate this further and validate the relative behavior predicted by the Gustafsson limit, namely that a length-to-diameter ratio slightly less than 2 is optimum, we consider several other configurations where the length-to-diameter ratios of the antennas are varied relative to the previous configurations. These are illustrated in Figures 11 and 12.

The antenna configurations M1A and M2A are simply multiple arm folded dipole versions of the meander line M1. With an increase in the number of folded arms, there is a decrease in the antenna's length-to-diameter ratio and an increase in the antenna's radiation resistance, exactly as occurs with the folded spherical and cylindrical helix antennas.

The antenna configurations M3A and M4A are also derived from meander line antenna M1. They have a decreased width and an increased length. These changes obviously translate into an increase in the antenna's length-to-diameter ratio.

A comparison of the performance properties of all these antennas is presented in Table 2. The ratio of Q/Q_{lb} for each antenna is presented and compared to the Gustafsson limit is Figure 13.



M1A

M2A

Figure 11. Electrically small planar antennas with decreased length-to-diameter ratios – M1A and M2A.

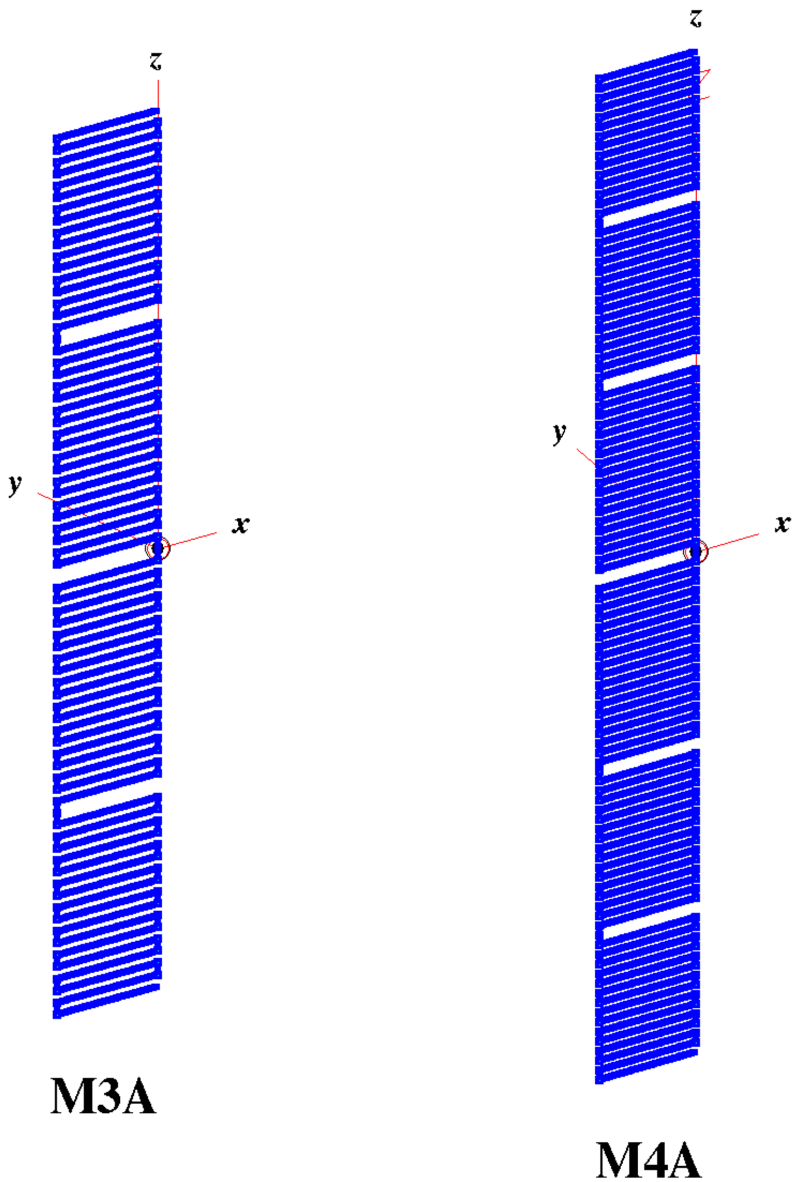


Figure 12. Electrically small planar antennas with increased length-to-diameter ratios - M3A and M4A.

As expected, the ratio of Q/Q_{lb} increases with either an increase or decrease in length-to-diameter ratio, illustrating that for optimum bandwidth, the electrically small planar antenna should have a length-to-diameter ratio slightly less than 2.

Table 2. Performance comparison of the electrically small planar antennas, M1A, M2A, M3A, and M4A.

Antenna Configuration	Frequency (MHz)	Overall Height (λ)	*ka*	*l/d*	Radiation Resistance (Ω)	Radiation Efficiency (%)	*Q*	Q/Q_{lb}
M1	329.7	0.088	0.308	1.99	3.1	63.7	166.8	4.44
M1A	338.3	0.089	0.453	0.796	13.4	78.5	82.8	6.40
M2A	346.3	0.092	0.849	0.362	51.4	87.5	43.7	15.56
M3A	341	0.168	0.531	7.37	9.5	85.9	53.12	6.21
M4A	260.1	0.18	0.567	10.35	10.7	85.1	50.76	7
CSTM4A	281.9	0.19	0.614	10.35	14.4	-	31.2	5.25

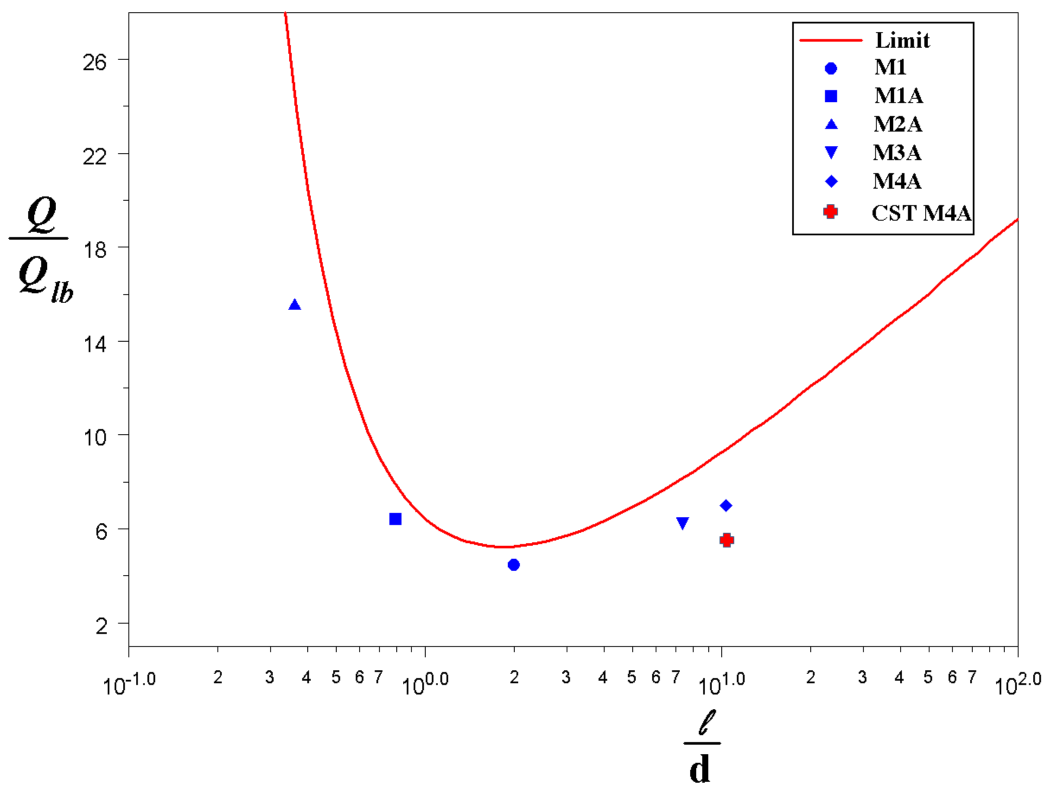


Figure 13. Comparison of the ratio Q/Q_{lb} for the electrically small planar antennas (M1A through M4A) relative to the Gustafsson limit.

We note however, that the Q/Q_{lb} ratios for the antennas are much lower than the Gustafsson limit causing us to question the validity of the simulations or possibly the statement of the limit. We have communicated our concern to Gustafsson and are discussing the matter at the time of writing. To validate the simulations to some extent, we modeled the M4A antenna configuration in CST's Microwave Studio and obtained similar results. The Q/Q_{lb} ratio determined from the Microwave Studio simiulations is depicted in Figure 13.

While our investigation of this topic is ongoing, we are confident that the behavior predicted by the Gustafsson limit is valid in that optimum length-to-diameter ratio for the electrically small planar antenna is slightly less than 2.

Discussion

We have modeled and compared the performance properties of several electrically small planar antennas for the purpose of comparing their quality factors to the lower bound and the Gustafsson limit. Our primary objective is to utilize these comparisons for the purposes of optimizing the bandwidth of electrically small planar antennas. We emphasize at this time that we are only considering small antennas that exhibit a single resonance within their defined operating bandwidth.

As expected from the Gustafsson limit, the optimum bandwidth of the small planar antenna is a function of its length-to-diameter ratio. We have also demonstrated that, in some cases, it is somewhat trivial to impedance match these antennas using near field reactive coupling or shunt stub parallel inductor. We note that efficient matching often becomes more of a challenge at lower frequencies and smaller antenna sizes.

Future work in this area is taking into account that many planar antennas operate in conjunction with a dielectric substrate which has the effect of lowering the antenna's operating frequency. We are additionally considering planar antennas that operate against in-line ground planes. In these applications, the antenna size is not limited to the antenna geometry but must include some portion of the ground plane as it is often the dominat source of radiation.

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