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Theory of Frequency-Independent Antennas as Traveling-Wave Antennas And Their Asymptotic Solution by Method of Stationary Phase

Johnson J. H. Wang
Wang Electro-Opto Corporation (WEO)
2140 Newmarket Parkway, Suite 110
Marietta, Georgia 30067 USA
Email: jjhwang@weo.com

I. Introduction

The theory of frequency-independent (FI) antennas has not been rigorously developed. The recent article by Mushiake [1] correctly pointed out that "the origin of the broadband characteristics of those antennas is in their shapes derived from the self-complementary antennas, rather than their log-periodic shapes." A recent symposium paper by this author [2] pointed out the shortcomings of existing theories on FI antennas, and gave a brief overview on the physical foundation of a class of FI antenna, the spiral-mode microstrip antennas. The approach was based on the traveling-wave (TW) antenna concept, which had been initially formulated in an earlier paper [3].

Specifically, fundamental elements are missing in existing theories on FI antennas. The only concrete element of the existing theory in the literature appears to be the real constant impedance for self-complementary antennas. Yet even this theory is not adequate for real-world FI antennas due to its assumption of infinite dimensions for the antenna. Theories based on the log-periodic or equiangular features, though intuitively plausible, do not add much useful insights into the design of FI antennas and are sometimes misleading. For example, speculative and misleading remarks were made in applying existing theories to the log and Archimedean spiral antennas, culminating in the statement that "the Archimedean spiral is by far the most popular of the two types of spirals mostly because the government funding of R&D programs was concentrated on the Archimedean spirals" [4].

This paper presents a TW theory for practical FI antennas as well as an asymptotic solution by the method of stationary phase, which can be applied to the design of real-world FI antennas in a simple and practical manner. Although the detailed analysis here is on two-arm spiral antennas fed with a 180° balun (mode-1 excitation) [3], the theory and technique are applicable to other planar FI antennas and for other modes. For example, the application to an FI omnidirectional mode-0 spiral-mode microstrip antenna [3] will be discussed in a separate paper [5].

II. Formulation of Planar FI Antennas as a TW Antenna

The FI antenna under consideration is depicted in Fig. 1, a planar TW antenna with a backing ground plane. The use of spherical, cylindrical and rectangular systems with (r, θ, ϕ), (ρ, ϕ, z) and (x, y, z) coordinates, respectively, is implicit, with the z-axis being normal to the ground plane. Without loss of generality, and in view of the reciprocity theorem, we consider only the transmit case.

It is assumed here that a TW wave has been successfully launched — by having a self-complementary planar structure S or by some other means yet to be discovered. Under this assumption one merely has to treat it as a radiation problem with equivalent surface currents that can be readily derived from a simple TW theory in closed form since the source and fields are sufficiently decoupled. This approach enables us to circumvent the mathematical complexities associated with these problems, which are generally formulated by quasi-Green's-functions using Sommerfeld integrals in the spectral domain [6]. Approximate antenna properties, including gain pattern and impedance, can be obtained under the assumption of TW wave as depicted in Fig. 1.

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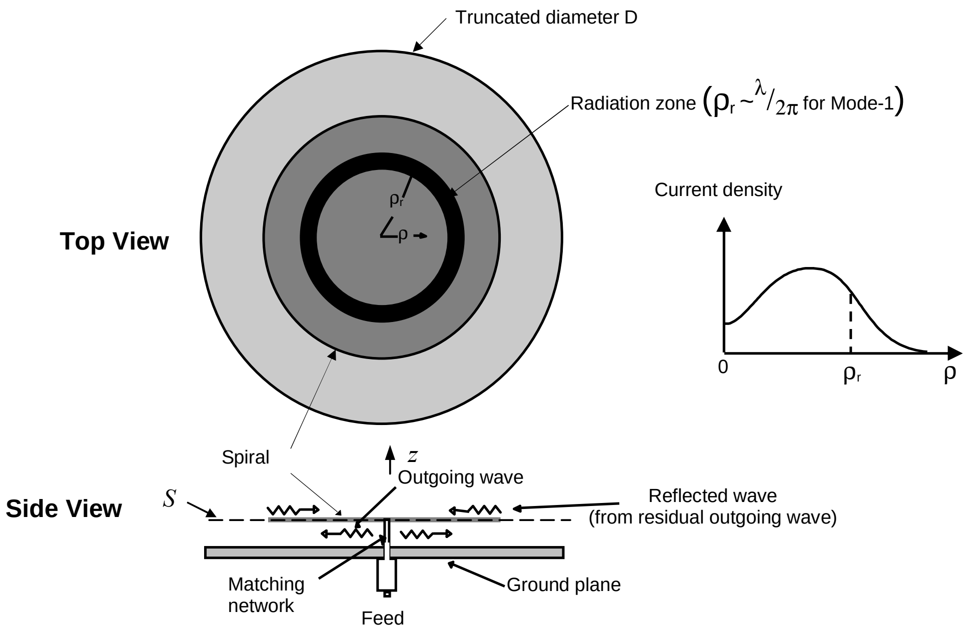

Figure 1. The radiation and radiation zone of a TW antenna.

As a result of the extensive research in planar antennas such as the microstrip antennas, simple formulations easy to evaluate and directly relating radiation to source have been established. Under the assumption of the TW antenna model in Fig. 1, radiation properties can be evaluated and interpreted by the following representation for the electric field **E** at **r** in the far-field [6, p.129 and Chapter 8].

$$\mathbf{E(r)} = \frac{jk\eta \exp(-jkr)}{4\pi r} \int_S \left[\left\{ J_\theta \hat{\theta} + J_\phi \hat{\phi} \right\} \exp(jk\hat{r} \bullet \mathbf{r}') \right] \mathrm{d}s' \qquad (1)$$

where **r** and **r′** are field and source position vectors, respectively; and the symbol "ˆ" over a variable signifies a unit vector.

The equivalent electric current ***J*** consists of only θ and ϕ components perpendicular to the field vector ***r***. Thus $J_\theta = J_\rho$ in polar coordinates on *S*. The planar surface of the antenna, *S*, is initially assumed to be infinite. It will be shown in the next section that, in a properly designed FI antenna, radiation takes place efficiently in a narrow circumferential radiation zone ΔS. As a result, by the method of stationary phase, the domain of the integral can be effectively reduced from *S* to ΔS. Physically, this allows truncation of the antenna to practical dimensions only slightly larger than those prescribed by the radiation zone.

III. Asymptotic Solution by the Principle of Stationary Phase

Let *I* denote the integrand of Eq. (1) for either θ or ϕ component, we have

$$I = J \exp(j\psi_J + jk\hat{r} \bullet \mathbf{r}') = J \exp(j\psi_J + jk\hat{r} \bullet \rho') \qquad (2)$$

where ψ_J and *J* denote, respectively, the phase and amplitude of either the θ or the ϕ component of the surface current ***J***, which now has only ρ and ϕ components. A two-arm mode-1 spiral antenna, or other planar FI 2-arm antennas, is assumed here.

To achieve stationary phase in the circumferential radiation zone ΔS, ψ_J must be stationary versus ρ with respect to the adjacent arms, the nearest being the arm of the other spiral, and the next one being that of the same spiral. **Since on and within a spiral arm the phase is essentially constant with respect to ρ, a stationary phase versus ρ between adjacent spiral arms can only be realized if the phase between them is $2n\pi$, where n = 0, 1, 2,… but realistically 1. This condition can be met if the currents in 4 adjacent spiral arms, 2 on either side, including both spirals, are phase stationary in the radiation zone.**

With the two spirals fed 180° apart, phase stationary dictates that: (1) The radiation zone be located at $\rho = \lambda/(2\pi)$ (λ being the wavelength of the TW), and (2) the phase change to the adjacent arm (of the other spiral) equals π. Specifically, the change in phase, $\Delta\psi_J$, with respect to ρ, from (ρ, ϕ) to the adjacent arms must satisfy the following two conditions:

$$\Delta\psi_J = 2\pi \text{ between } \phi \text{ and } (\phi + 2\pi) \text{ on the same spiral} \qquad (3a)$$
$$\Delta\psi_J = \pi \text{ at } \phi \text{ between adjacent arms (but of different spirals)} \qquad (3b)$$

IV. Mode-1 Archimedean Spiral

The center lines of a two-arm Archimedean spiral on the S plane are given by

$$\rho_1 = a\phi, \qquad \phi \in [\phi_0, \phi_t] \qquad (4a)$$
$$\rho_2 = a(\phi - \pi), \qquad \phi \in [\phi_0 + \pi, \phi_t + \pi] \qquad (4b)$$

The two feed points are at $\rho = \pm d/2$ (or $\phi = \phi_0$ and $\phi_0 + \pi$) for arms #1 and #2, respectively. ϕ_t and ($\phi_t + \pi$) are the coordinates of the terminals of spiral #1 and spiral #2, respectively.

The phase change $\Delta\psi_J$ of the component current as it travels along a spiral arm is determined by the phase velocity of the TW and the distance traveled. The arc lengths L_1 along spiral arm #1 from its feed point to (ρ, ϕ), and L_2 along the adjacent spiral arm #2 from its feed point to (ρ_2, ϕ) (note here by choice $\rho_2 > \rho$), respectively, are given by

$$L_1 = \frac{a}{2}\left[\phi\sqrt{1+\phi^2} + \sinh^{-1}\phi\right]_{\phi_0}^{\phi} \qquad (5a)$$

$$L_2 = \frac{a}{2}\left[\phi\sqrt{1+\phi^2} + \sinh^{-1}\phi\right]_{\phi_0}^{\phi-\pi} \qquad (5b)$$

L_1 and L_2 are measured along the center line of each spiral arm. For the Archimedean spiral antenna, there are two important relationships between adjacent (but different) arms of spirals

$$\Delta\rho \equiv a\phi - a(\phi - \pi) = a\pi \qquad (6a)$$
$$\Delta L \equiv (L_1 - L_2) \sim \rho\Delta\rho/a \qquad \text{as } |a| \to 0 \qquad (6b)$$

Therefore, conditions (3a) and (3b) are both satisfied at $\rho = \lambda/(2\pi)$ as long as a is small. As a result, a mode-1 Archimedean spiral antenna with reasonably tight winding can be easily truncated. And its radiation properties can be obtained by using the method of stationary phase with the domain of integration, S, in Eq. (1) replaced by an appropriately chosen radiation zone, ΔS, which is the circumferential area having about 5 adjacent spiral arms centered at ρ.

V. Mode-1 Log Spiral

For a log spiral antenna on the S plane, the center lines of the two spirals are given by

$$\rho_1 = \exp(a\phi), \qquad \phi \in [\phi_0, \phi_t] \qquad (7a)$$
$$\rho_2 = \exp[a(\phi - \pi)], \qquad \phi \in [\phi_0 + \pi, \phi_t + \pi] \qquad (7b)$$

The two feed points are at $x = \pm d/2$, or $\phi = \phi_0$ and $\phi_0 + \pi$ for arms #1 and #2. The arc lengths L_1 along spiral arm #1 from its feed point to (ρ, ϕ), and L_2 along the adjacent spiral arm #2 from its feed point to (ρ_2, ϕ) (note here $\rho_2 > \rho$ by choice), respectively, are given by

$$L_1 = \sqrt{1+\frac{1}{a^2}}[e^{a\phi} - e^{a\phi_0}] \tag{8a}$$

$$L_2 = \sqrt{1+\frac{1}{a^2}}[e^{a(\phi-\pi)} - e^{a\phi_0}] \tag{8b}$$

Thus,

$$\Delta L \equiv L_1 - L_2 = \sqrt{1+\frac{1}{a^2}}[1 - e^{a\pi}]\rho \tag{9}$$

Condition (3a) can be satisfied at $\rho = \lambda/(2\pi)$ if $|a| \to 0$. However, condition (3b) dictates that a not only be small in magnitude but also satisfy

$$\Delta L = \lambda/2 \qquad \text{at } \rho = \lambda/(2\pi) \tag{10}$$

The requirement of Eq. (10) adds considerable difficulties to the design of log spiral antennas that must tradeoff between performance requirements and size truncation. This is a problem not encountered in the Archimedean spiral antenna.

VI. Conclusions

The theory presented here has been found to be consistent with experimental data, with better agreement than some results in the literature obtained by brute-force numerical computation. More importantly, the technique is useful for design and synthesis of FI antennas because of its simplicity and its direct and close relevance to the physical parameters and performance of the FI antenna. The advantage of the Archimedean spiral antenna over the log spiral antenna, contrary to the observation in [4] and others in the literature, is pointed out and demonstrated based on this theory.

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