APPENDIX

C

THE STORY OF THE "MAXWELL" EQUATIONS

The story of the Maxwell equations is fascinating in many ways.†

First, as we shall see, the equations should be called, more appropriately, the Heaviside equations.

Second, the story covers less than the fourth quarter of the nineteenth century.

Third, there were few *Maxwellians*, the main actors being Oliver Heaviside (1850–1925), who was self-taught, George Francis FitzGerald (1851–1901), who worked at Trinity College in Dublin, Oliver Lodge (1851–1940), who was at University College in Liverpool, and Heinrich Hertz (1857–1894), who was an experimental physicist at the Karlsruhe Physical Institute. John-Henry Poynting (1852–1914) contributed a major concept when he proposed his theorem in 1884, but Heaviside discovered it practically at the same time. James Clerk Maxwell (1831–1879) was an actor through the publication in 1873 of his *Treatise on Electricity and Magnetism*. At the time of his death, he was working on a second edition of the *Treatise*.

Fourth, all the actors were British, except for Hertz, who was German. Maxwell, who was Scottish, became Professor of Experimental Physics at Cambridge but, after his death, there were no Maxwellians at Cambridge, and none at Oxford.

Fifth, the main actor was Heaviside, who had to leave school when he was sixteen. He started life as a telegrapher in Newcastle, where he stayed for eight years, but had no real job for the rest of his life. He was a prolific writer, despite the fact that he had poor health and lived as a recluse, partly by temperament, partly because of his deafness, and partly because of his great poverty. For many years he published his theories in *The Electrician*, which was a trade journal. For this he was paid 40 pounds per year, which was less than the salary of a laborer. Near the end of his life he received a modest government pension of 120 pounds per year, but remained exceedingly poor and miserable, despite the many honors that were bestowed upon him. Although he was self-taught, he had become one of the best, if not *the* best, physicist-mathematician of his time.

Maxwell's *Treatise* is unreadable. Hunt (1991), who clearly regrets having spent so much time on it, says that it is rambling, obscure, inconsistent, contradictory, awkward, confusing, and on some points simply wrong; that it has no clear focus, no or-

†The best reference is Hunt (1991), but see also Bork (1963), Heaviside (1950 and 1971), Gillispie (1980), and Harman (1998).

derly presentation, and that the mathematics is clumsy and involved! It is a wonder that the Maxwellians managed to master it. Maxwell had great admiration for Faraday, and he was attempting to put Faraday's ideas in mathematical form.

The *Treatise* does *not* state the Maxwell equations because Maxwell reasoned mostly in terms of the potentials V (then called ψ) and $\boldsymbol{A}$. Shifting from V and $\boldsymbol{A}$ to $\boldsymbol{E}$ and $\boldsymbol{B}$ caused much debate about "the murder of ψ." The potentials then came to be considered as mathematical fictions.

Heaviside focused on the electromagnetic *field*, like Faraday and Maxwell. His main motivation was to improve signaling on submarine cables. He finally recast the Maxwell theory in the form of the four vector "Maxwell" equations in 1884, in the same year that Hertz stated them in Cartesian form. Hertz admitted that Heaviside had the priority, in view of the fact that his own "proof" was "very shaky." At the time, vector notation seemed to everyone to be highly esoteric.

Modern Physics owes very much to Heaviside. He was the first to apply vector analysis as we know it today to electromagnetic fields, and the first to use *rationalized* units. (With unrationalized units, factors of 4π appear in most of the important equations.) He discovered the *Poynting* theorem the same year as Poynting. He proposed the existence of the Heaviside layer† and developed Operational Calculus. He also discovered the "Lorentz" force $Q\boldsymbol{v} \times \boldsymbol{B}$ fifteen years before Hendrik Lorentz (1853–1928). As early as 1888 he calculated the $\boldsymbol{E}$ and $\boldsymbol{B}$ fields of a moving charge (Sec. 13.2). That was a most important result because it showed how the field contracts lengthwise and that *the contraction involves the factor γ of relativity!* This phenomenon was later called the *FitzGerald contraction* and served to explain the negative result of the Michelson-Morley experiment.

†This is a region in the upper atmosphere where the degree of ionization varies with altitude in such a way that low-frequency radio waves are reflected downward. At TV frequencies, waves bend slightly and escape.