

# Understanding Noise Figure

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One of the most frequently discussed forms of noise is known as *Thermal Noise*. Thermal noise is a random fluctuation in voltage caused by the random motion of charge carriers in any conducting medium at a temperature above absolute zero ( $K=273 + \text{Celsius}$ ). This cannot exist at absolute zero because charge carriers cannot move at absolute zero. As the name implies, the amount of the thermal noise is to imagine a simple resistor at a temperature above absolute zero. If we'll use a very sensitive oscilloscope probe across the resistor, we'll see a very small AC noise being generated by the resistor.

The RMS voltage is proportional to the temperature of the resistor and how resistive it is.

**Larger resistances and higher temperatures generate more noise.**

The formula to find the RMS thermal noise voltage of a resistor is:

$$V_n = \sqrt{4kTRB}$$

where:

k = Boltzman constant ( $1.38 \cdot 10^{-23}$  Joules/Kelvin)

T = Temperature in degrees Kelvin ( $K= +273 \text{ Celsius}$ )

R = Resistance in ohms

B = Bandwidth in Hz in which the noise is observed (RMS voltage measured across the resistor is also function of the bandwidth in which the measurement is made).

As an example, that 100 k $\Omega$  resistor with 1MHz bandwidth will add noise to the circuit as below:

$$V_n = (4 \cdot 1.38 \cdot 10^{-23} \cdot 300 \cdot 100 \cdot 10^3 \cdot 1 \cdot 10^6)^{1/2} = 40.7 \mu\text{V RMS}$$

Low impedances are desirable in low noise circuits. For an easier job we may actually measure the noise of a device in a 1MHz bandwidth (it is easier), we usually convert the number to 1Hz bandwidth (the lowest band denominator) in order to compare it to other sources.

Noise Bandwidth, B, is defined as the equivalent rectangular pass-band that passes the same amount of noise power as is passed in the usable receiver band, and that has the same peak in-band gain as the actual device has. It is the same as the integral of the gain of the device over the usable frequency bandwidth.

Typically, B is approximately equal to the 3 dB bandwidth. For best sensitivity, B should be no greater than required for the information bandwidth.

In RF applications, we usually deal with circuits having matched input and output impedances, and are therefore more concerned with the power available from a device than the voltage. In this case, it is common to express the noise of a device in terms of the available noise power. The Maximum Power Transfer Theorem predicts that the noise power delivered from a source to a matched load can be delivered:

$P = (V_{oc}/2)^{2/R} = kTB = -174\text{dBm/Hz}$  (the reference noise level in a 1Hz bandwidth, at room temperature)

- Temperatures correspond to power levels. When the temperature of a resistor is doubled the power output from it is doubled (the voltage is proportional to the square root of the temperature).

- Powers from uncorrelated sources are additive so noise temperatures are additive. In addition to thermal noise, amplifiers and other devices with semiconductors in them also contribute other forms to signal.

## Noise Figure

To characterize the receiver alone, [Harald T. Friis](#) introduced in 1944 the Noise Figure (NF) concept which characterized the degradation in Signal to Noise Ratio (SNR) by the receiver.

- Noise Figure (NF) is a measure of how much a device (such an amplifier) degrades the Signal to Noise ratio (SNR).

$$\begin{aligned} SNR_{input[linear]} &= Input\_Signal_{[Watt]} / Input\_Noise_{[Watt]} \\ SNR_{input[dB]} &= Input\_Signal_{[dB]} - Input\_Noise_{[dB]} \end{aligned}$$

$$\begin{aligned} SNR_{output[linear]} &= Output\_Signal_{[Watt]} / Output\_Noise_{[Watt]} \\ SNR_{output[dB]} &= Output\_Signal_{[dB]} - Output\_Noise_{[dB]} \end{aligned}$$

- Noise Factor (linear not dB) of a receiver is the ratio of the SNR at its input to the ratio of the SNR at its output.

$$\begin{aligned} NoiseFactor\_F_{(linear)} &= SNR_{input[linear]} / SNR_{output[linear]} \\ NoiseFactor\_F_{[dB]} &= SNR_{input[dB]} - SNR_{output[dB]} \\ NoiseFigure\_NF_{[dB]} &= 10*LOG (NoiseFactor\_F_{(linear)}) \end{aligned}$$

- Note that **SNR at the output will always be smaller than the SNR at the input**, due to the fact that circuits always add to the noise in a system.
- The Noise Factor, at a specified input frequency, is defined as the ratio of the total Noise Power per unit bandwidth available at the output port when noise temperature of the input termination is standard (290 K) to that portion of engendered at the input frequency by the input termination.

$$NoiseFactor\_F = (Available\_Output\_Noise\_Power) / (Available\_Output\_Noise\_due\_to\_Source)$$

The maximum Noise Figure of the receiver when is given the required Sensitivity and the required Bandwidth:

$$Receiver\_Noise\_Figure_{[dB]} = 174 + Receiver\_Sensitivity_{[dBm]} - 10*LOG(BW_{[Hz]}) - SNR_{[dB]}$$

As can be seen from the formula above, narrow Bandwidth and smaller SNR will relax the required receiver Noise Figure requirements.

- When designing circuits for use with extremely weak signals, noise is an important consideration. The noise contribution of each device in the signal path must be low enough that it will not significantly degrade the Signal to Noise Ratio.
- Noise found in a microwave system can be generated from external sources, or the system itself.

- The Noise level of a system sets the lower limit on the magnitude of a signal that can be detected in the presence of the noise. So, to achieve the best performance you need to have a minimum residual noise level.
- Noise Figure is used to describe the noise contribution of a device. An ideal amplifier would have no noise of its own, but would simply amplify what went in to it.

For example a 10dB amplifier would amplify the Signal (and the Noise) at its input by 10dB. Therefore, although the noise floor at the output of the amplifier would be 10dB higher than at the input.

- The "ideal noiseless" amplifier would not change the Signal to Noise ratio (SNR).
- A "real world" amplifier will not amplify only the noise at its input, but will contribute its own noise to signal. This reduces the Signal to Noise ratio at the output of the amplifier.
- So, the "real world" amplifier has two major internal components: an "ideal noiseless" amplifier and a noise source. The noise source adds noise to any signal what enters to the amplifier and then the ideal amplifier amplifies the whole thing by an amount equal to its gain, with no noise contribution of its own.

For example a 10dB attenuator placed at the input of an amplifier will increase the total Noise Figure of the system with 10dB.

- An attenuator placed at the input of a system will increase the total Noise Figure with the same amount of its attenuation.
- An attenuator placed at the input of a receiver would not affect the SNR if the level at its output is inside of the input dynamic range of the receiver.

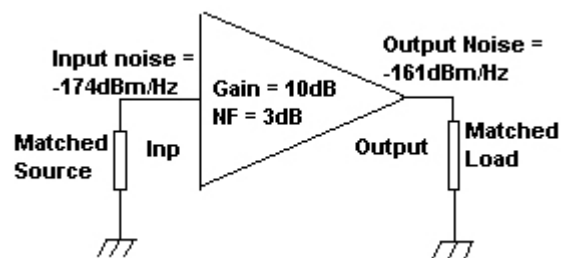
As an example let's assume that we have an amplifier at room temperature with 10dB of gain which has only a matched resistor at its input and output.

The noise at the input of the amplifier must be -174dBm/Hz.

If the amplifier is known to have a 3dB NF, the internal noise source adds an equal noise to the input noise before amplification.

Then 10dB of gain increases the noise by 10dB.

Therefore, the noise at the output of the amplifier is 13dB higher than at the input, or  $(-174\text{dBm/Hz} + 10\text{dB gain} + 3\text{dB NF}) = -161\text{dBm/Hz}$ .



- The noise contribution of the amplifier's noise source is fixed and does not change with input signal.
- Therefore, when more noise is present at the amplifier input, the contribution of the internal noise source is less significant in comparison.

- When the noise into an amplifier is higher than kTB (-174dBm/Hz), the amplifier's Noise Figure plays a smaller role in the amplifier's noise contribution.
- The Noise Figure (NF) of a device is only calculated with the input noise level at kTB.
- The Noise Temperature at the output of an amplifier is the sum of the Noise Temperature of the source and the Noise Temperature of the amplifier itself multiplied by the Power Gain of the amplifier.

$$T_{out} = G * ( T_{ampl} + T_{source} )$$

$T_{out}$  = Noise Temperature at amplifier output in degrees Kelvin.

$G$  = Power Gain in linear scale not in dB.

$T_{ampl}$  = Noise Temperature of amplifier.

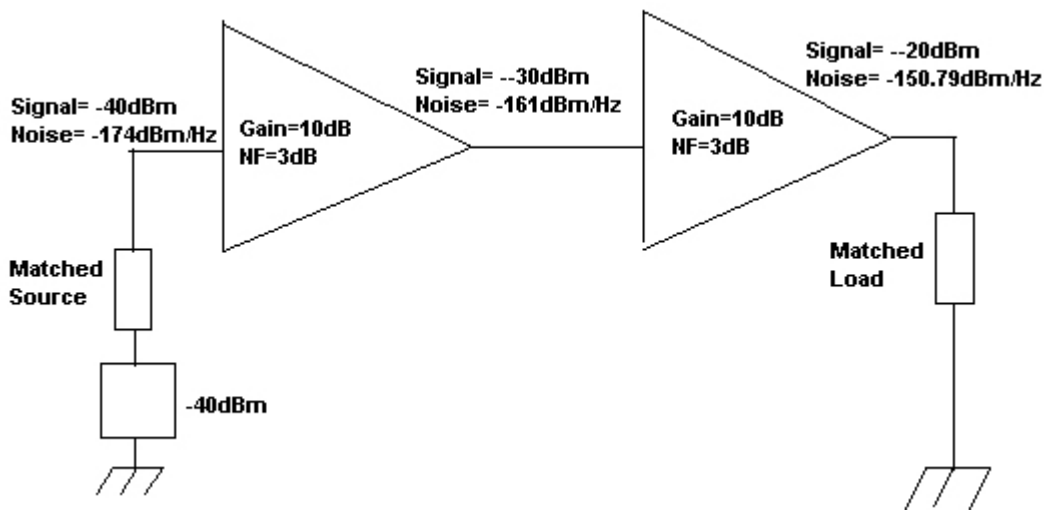
$T_{source}$  = Noise Temperature of source.

The same formula is valid for attenuators.

$$T_{out} = G_{att} * ( T_{att} + T_{source} )$$

- The Noise Figure of an attenuator is the same as the attenuation in dB.
- The Noise Figure of an attenuator preceding an amplifier is the Noise Figure of the amplifier plus the attenuation of the attenuator in dB.

If we use cascaded amplifiers:



For above example both amplifiers has 10dB gain and NF=3dB.

The signal goes in at -40dBm with a noise floor at kTB (-174dBm/Hz).

We can calculate that the signal at the output of the first amplifier is -30dBm and the noise is: (-174dBm/Hz input noise) + (10dB of gain) + (3dB NF) = -161dBm/Hz.

Let see how many kTBs are entering in the second amplifier:

(-161dBm/Hz) is 13dB greater than kTB (-174dBm).

13dB is a power ratio of 20x. So, the noise floor at the second amplifier is 20 times kTB or 20kTB.

Next calculate how many kTBs are added by the noise source of the second amplifier (in this case, 1kTB because the NF=3dB).

Finally calculate the increase in noise floor at the second amplifier as a ratio and convert to dB.

Ratio of (input noise floor) + (added noise) to (input noise floor) is:

$$(20kTB+1kTB) / (20kTB) = 20/21$$

$$\text{In dB} = 10\text{LOG} (21/20) = 0.21\text{dB}$$

Therefore, the second amplifier only increases the noise floor by 0.21dB even though it has a noise figure of 3dB, simply because the noise floor at its input is significantly higher than kTB. The first amplifier degrades the signal to noise ratio by 3dB, while the second amplifier degrades it only 0.21dB.

- When amplifiers are cascaded together in order to amplify very weak signals, it is generally the first amplifier in the chain which will have the greatest influence upon the signal to noise ratio because the noise floor is lowest at that point in the chain.

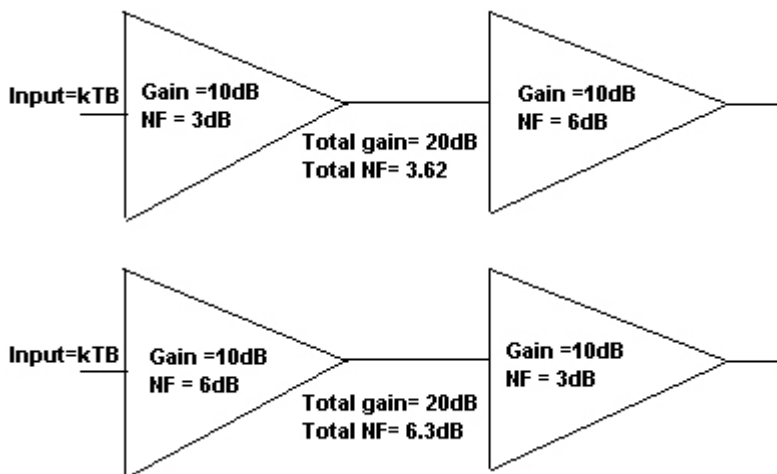
Determining the total Noise Figure of a chain of amplifiers (or other devices):

$$NFactor\_total = NFact1 + (NFact2-1)/G1 + (NFact3-1)/(G1*G2) + (NFact3-1)/(G1*G2*....Gn-1)$$

where  $NFactor = \text{Noise factor of each stage (Linear not in dB)}$ .

$$\text{Noise Figure}_{[dB]} = 10*LOG(NFactor)$$

$G = \text{Gain of each stage as a ratio, not dB (for example 4x, not 6dB)}$



The first amplifier in a chain has the most significant effect on the total noise figure than any other amplifier in the chain. The lower noise figure amplifier should usually go first in a line of amplifiers (assuming all else is equal). If we have two amplifiers with equal gain, but with different noise figures. Assume 10dB gain in each amplifier. One amp is NF = 3dB and the other 6dB. When the 3dB NF amplifier is first in cascade, the total noise figure, the total NF is 3.62dB. When the 6dB amplifier is first, the total NF is 6.3dB. This also applies to gain.

- If two amplifiers have the same Noise Figure but different gains, the higher gain amplifier should precede the lower gain amplifier to achieve the best overall Noise Figure.

- The overall Noise Factor of an infinite number of identical cascaded amplifiers is:

$$NFactor\_total = 1 + M$$

$$\text{with: } M = (F-1) / [1 - (1/G)]$$

where:  $F$  is  $\text{Noise\_Factor}_{(linear)}$  of each stage, and  $G$  is  $\text{Gain}_{(linear)}$  of each stage.

$$\text{NoiseFigure\_total}_{[dB]} = 10*LOG (NFactor\_total_{(linear)})$$

## Noise Figure of a Device

- The Noise Figure of a device is the degradation in the Signal to Noise Ratio (SNR) as a signal passes through the device.

The equation of the Noise Figure of a device adopted by the IEEE (Institute of Electrical and Electronics Engineers) is:

$$\text{Device } NF_{[dB]} = 10 \cdot \text{LOG} [(N_a + k \cdot T_o \cdot B \cdot G) / (k \cdot T_o \cdot B \cdot G)]$$

$N_a$ =Added Noise Power [W],  $T_o$ =290°K,  $B$ =Bandwidth[Hz],  $G$ =Gain[Linear not dB],  $k$ =Boltzman ct.

Therefore, the Noise Figure of a device is the ratio of the total Noise Power at the output, to that portion of the Noise Power at the output due to noise at the input when the input source temperature is 290°Kelvin.

- The Noise Figure of a device is independent of the signal level as long as the device is linear (output power vs. input power).

## Equivalent Input Noise Temperature

Initially used in Satellite Receivers, the Equivalent Input Noise Temperature ( $T_e$ ), is used to describe the noise performance of a device rather than the Noise Figure.

- $T_e$  is the Equivalent Temperature of a source impedance into a perfect (noiseless) device that would produce the same Added Noise Power ( $N_a$ ).

$T_e$  is mostly used as a system parameter, and is defined as:

$$T_e = N_a / (k \cdot G \cdot B)$$

the relation with the Noise Factor (F) is:

$$T_e = T_o (F - 1)$$
$$F = 1 + (T_e / T_o)$$

For example a device with 0.5dB Noise Figure (Noise Factor,  $F = 1.12$ ), at  $T_o = 290k$ , would have an Equivalent Input Noise Temperature,  $T_e = 35.4k$

Resulting noise temperature referred to the input ( $T_{eq}$ ) of cascaded stages is given by:

$$T_{eq} = T_1 + (T_2 / G_1) + [T_3 / (G_1 \cdot G_2)] + \dots$$

Noise temperature (in Kelvin) of each component in the cascade is:  $T_{(1,2,3...)} = T_o (F_{(1,2,3...)} - 1)$

Power gain (Linear not dB) of each component in the cascade is:  $G_{1,2,3...}$

The Noise Figure of the cascade is:  $NF_{[dB]} = 10 \cdot \text{LOG} [1 + (T_{eq} / T_o)]$

- In Satellite and Space Receiving Systems, the noise level coming from the antenna can be very low, limited by the ground noise (due to the side-lobe radiation of the antenna), and by the background sky temperature (with values often below 100k). In these situations, small changes in the Noise Figure of the receiving system may result in much more change of the Signal to Noise Ratio (SNR).

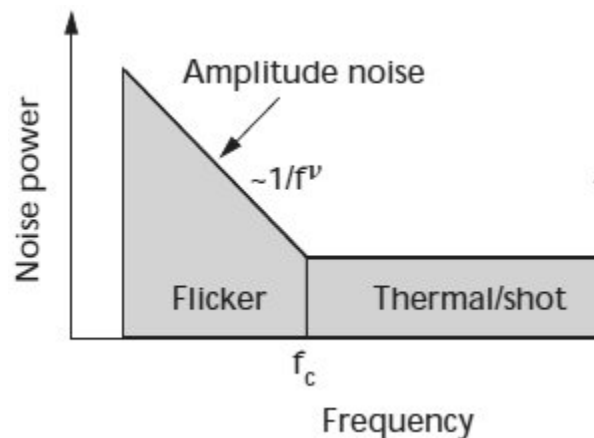
## Noise Figure of Other Devices

- All the devices which process a signal contribute noise and thus have noise figure.
- Amplifiers, Mixers, transistors, diodes, etc all have noise figures.
- For example, RF attenuators have a Noise Figure floor equal to their attenuation value. A 10dB pad has a 10dB NF. If a signal enters in a pad and the noise floor is at -174 dBm/Hz the signal is attenuated by 10dB while the noise floor remains constantly (it cannot get any lower than -174 dBm/Hz at room temperature). Therefore the signal to noise ratio through the pad is degraded by 10dB. Like amplifiers, if the noise floor is above kTB, the signal to noise ratio degradation of the pad will be less than its noise figure.
- The Radiation Resistance of the antenna does not convert power to heat, and so is not a source of thermal noise.
- The load impedance of the input of the receiver does not contribute directly to receive noise. Therefore, it is indeed possible, and even common, for a receiver to have a Noise Factor of less than 2x (or equivalently, a Noise Figure of less than 3dB).

## Types of Noise sources

There are several types of noise sources in electrical circuits. However, we discuss only three important noise sources here.

1. Thermal or [Johnson](#) - [Nyquist](#) Noise
2. Shot Noise
3. 1/f Noise (Also called Flicker or Pink noise)
4. White Noise
5. Burst Noise



### 1. Thermal Noise

- This is the noise generated by thermal agitation of electrons in a conductor.
- Also called *Johnson-Nyquist* Noise, is the random white noise (flat with frequency) generated by thermal agitation of electrons in a conductor or electronic device. It is produced by the thermal agitation of the charges in an electric conductor and is proportional to the absolute temperature of the conductor.
- It manifests itself in the input circuits of audio equipment such as microphone pre amps, or antenna input of a receiver, where the signal levels are low.
- The Thermal Noise level is the limiting minimum noise any circuit can attain at a given temperature.

Note that thermal Noise Power, per Hertz, is equal throughout the frequency spectrum, depends only on  $k$  and  $T$ .

- Thermal noise in the resistance of the signal source is the fundamental limit on achievable signal sensitivity.
- Thermal Noise has a Gaussian amplitude distribution in the time domain and is evenly distributed across the spectrum.
- Thermal noise's spectral breadth and its sources' ubiquity lead it to dominate other noise types in many applications.

## 2. Shot Noise

- Shot Noise normally occurs when there is a potential barrier (voltage differential).
- PN junction diode is an example that has potential barrier. When the electrons and holes cross the barrier, Shot Noise is produced.
- For example, a diode, a transistor, and vacuum tube, all will produce Shot noise.
- A junction diode will typically have two components of noise. One is Thermal Noise, and the other is Shot Noise.
- Note that if the active device provides amplification, the noise also gets amplified along with the signal.
- On the other hand, a resistor normally does not produce Shot Noise since there is no potential barrier built within a resistor. Current flowing through a resistor will not exhibit any fluctuations. However, current flowing through a diode produces small fluctuations. This is due to electrons (in turn, the charge) arriving in quanta, one electron at a time. The current flow is not continuous, but limited by the quantum of the electron charges.
- Shot Noise is proportional to the current passing through the device.
- Shot Noise characteristic is white.

## 3. Flicker Noise - $1/f$ (one-over-f) Noise

- Flicker Noise is found in many natural phenomena such as nuclear radiation, electron flow through a conductor, or even in the environment. In electrical engineering, it is called also  $1/f$  (one-over-f) Noise.
- Flicker Noise is associated with crystal surface defects in semiconductors and is also found in vacuum tubes due to the oxide coating on the cathode.
- The noise power is proportional to the bias current, and, unlike Thermal and Shot Noise, Flicker Noise decreases with frequency. An exact mathematical model does not exist for flicker noise because it is so device-specific. However, the inverse proportionality with frequency is almost exactly  $1/f$  for low frequencies, whereas for frequencies above a few kilohertz, the noise power is weak but essentially flat.
- Flicker Noise is essentially random, but because its frequency spectrum is not flat, it is not a white noise. It is often referred to as pink noise because most of the power is concentrated at the lower end of the frequency spectrum.
- Flicker Noise is more prominent in FETs (smaller the channel length, greater the Flicker Noise), and in bulky carbon resistors. The objection to carbon resistors mentioned earlier for critical low noise applications is due to their tendency to produce flicker noise when carrying a direct current. In this connection, metal film resistors are a better choice for low frequency, low noise applications.
- Flicker Noise is usually defined by the corner frequency  $f_c$ , point where Flicker Noise is equal with White Noise.



- Under “typical” operating conditions, precision bipolar processes offer the lowest 1/f corners: around 1Hz to 10Hz.
- The corner for devices fabricated in high-frequency bipolar processes is often 1Hz to 10kHz.
- The 1/f corner frequency in MOSFETs goes as the reciprocal of the channel length, with typical values of 100kHz to 1MHz, and even up to 1GHz for latest nano-meter channel length processes.
- Devices built on III-V processes, such as GaAs FET's and Indium-Gallium-Phosphorous HBT, offer extremely wide bandwidths but yield higher frequency 1/f corners in the region of 100 MHz.

#### **4. White Noise**

- White Noise is the noise that has constant magnitude of power over frequency.
- Examples of White Noise are Thermal Noise, and Shot Noise.

#### **5. Burst Noise**

- Burst Noise or Popcorn Noise is another low frequency noise that seems to be associated with heavy metal ion contamination. Measurements show a sudden shift in the bias current level that lasts for a short duration before suddenly returning to the initial state. Such a randomly occurring discrete level burst would have a popping sound if amplified in an audio system.
- Like Flicker Noise, Popcorn Noise is very device specific, so a mathematical model is not very useful.
- However, this noise increases with bias current level and is inversely proportional to the square of the frequency  $1/f^2$ .

### **Noise reduction strategies**

Noise is a serious problem, especially where low signal levels are experienced, there are a number of common sense approaches to minimize the effects of noise on a system.

In this section we will examine several of these methods.

- Keep the source resistance and the amplifier input resistance as low as possible. Using high value resistances will increase thermal noise voltage.
- Total thermal noise is a function of the bandwidth of the circuit. Therefore, reducing the bandwidth of the circuit to a minimum will also minimize noise. There is also a requirement to match the bandwidth to the frequency response required for the input signal.
- Prevent external noise from affecting the performance of the system by appropriate use of grounding, shielding and filtering.
- Use a Low Noise Amplifier (LNA) in the input stage of the system.
- For some semiconductor circuits, use the lowest DC power supply potentials that will do the job.
- At the transistor level, device noise can be sensed and reduced with negative feedback. Current fluctuations in the transistor contribute to phase and amplitude noise. An unbypassed emitter resistor ( $R_E \sim 10..30$  ohms) reduces noise, but further noise improvement is achieved by sensing the emitter current and feeding back a signal to the base terminal. Successful PM and AM noise reduction of 20 dB has been demonstrated.

- At higher frequencies, the feedback capacitance of the device couples the shot noise of the base/collector junction (BJT) or thermal noise of the channel resistance (MOSFET) to the input and contributes to frequency-dependent noise. Optimum noise matching is achieved with BJTs in applications requiring low source resistance, whereas MOSFETs become viable for high source resistance applications.

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