

# Zeroth-Order and $TM_{10}$ Modes in One-Unit Cell CRLH Mushroom Resonator

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**Abstract**— In this letter, the zeroth-order and  $TM_{10}$  modes of composite right/left-handed (CRLH) mushroom resonator are investigated with regards to the number of cells. It is shown that in addition to  $2N-1$  metamaterial resonance frequencies in an  $N$ -cell mushroom resonator, there is also a resonance which belongs to  $TM_{10}$  mode of the unit-cell. This  $TM_{10}$  mode is the fundamental mode of the patch which occurs at a frequency above metamaterial resonance frequencies. These two types of resonance frequencies yield  $2N$  resonances in an  $N$ -cell CRLH mushroom resonator. This is beneficial especially in one-unit cell case due to dual-band operation. To investigate this feature, three types of mushroom resonant antennas with different number of cells ( $N=3, 2, 1$ ) are designed and implemented. Based on theoretical investigation, a single-cell dual-band CRLH mushroom resonant antenna is proposed. At the zeroth-order mode (2.88GHz), the proposed antenna has a dimension of approximately  $0.19\lambda \times 0.09\lambda \times 0.014\lambda$  while achieving a gain and an efficiency of -0.82 dBi and 46%, respectively.

**Index Terms** —Composite right/left-handed (CRLH) resonator, multiband antenna, mushroom structure, zeroth-order resonant antenna (ZORA).

## I. INTRODUCTION

OVER the last decade, increasing demands for low profile multifunctional antennas have resulted in considerable interest in a research community of electromagnetic metamaterials (MTM). MTM was first realized using artificial structures made of wires and split ring resonators (SRRs) [1]. Meanwhile, some researchers proposed another type of MTMs known as composite right/left-handed (CRLH) structures which is based on an equivalent circuit approach [2]. Two of their main applications are in leaky wave and resonant antennas. Introducing zero and negative-order modes ( $n=0, -1, -2, \dots$ ) is the intriguing property of CRLH resonators to achieve multiband functionality and miniaturization. It is well-known that mushroom configuration [3], as a CRLH structure, provides left-hand propagation. Based on CRLH resonator theory, the number of resonances ( $2N-1$ ) is defined by the number of cells ( $N$ ) [2]. Consequently, a multiband characteristic is obviously present in a multi-cell

CRLH resonant structure. Although the word "MTM" is compatible with a periodic structure, in reality, one has to reduce the number of cells to have a compact resonator and there are several reports on a single unit antenna, which is referred to as metamaterial-inspired antenna [4]-[7]. Nevertheless, previous works did not provide a theory to delineate how a metamaterial-inspired antenna can show dual-band operation.

In this letter, the zeroth-order and  $TM_{10}$  modes of CRLH mushroom resonators with regards to the number of cells are investigated. It is shown that in an  $N$ -cell mushroom resonator, the  $TM_{10}$  mode which is defined by the length of radiating edges of the unit-cell is excited above  $2N-1$  metamaterial resonance frequencies. These two types of resonance frequencies lead to  $2N$  resonances in an  $N$ -cell CRLH mushroom resonator. This is practical especially in one-unit cell case due to dual-band operation with more compact size in comparison with multi cells resonators. To investigate this feature, three types of mushroom resonant antennas with different number of cells ( $N=3, 2, 1$ ) are designed and implemented. The resonance frequencies are extracted from simulations and compared with measurement results. The commercial CST Microwave Studio software is adopted for simulations.

## II. THEORY OF CRLH RESONATOR

In practical implementation, the CRLH equivalent circuit represents the most general possible MTM structure. When a CRLH transmission line (TL) is not terminated to a matched load, standing wave is produced and TL converts to a resonator. For a resonator of length  $l$  consisting of  $N$  unit cells with period  $p$ , resonance frequencies occur where the physical length of the resonator,  $l$ , is multiple of half a wavelength ( $\lambda$ ) or the electrical length is a multiple of  $\pi$ .

$$\beta_m l = \left( \frac{2\pi}{\lambda} \right) \left( \frac{m\lambda}{2} \right) = m\pi, m = 0, \pm 1, \dots, \pm N - 1 \quad (1)$$

Substituting  $l=Np$ :

$$\beta_m = \frac{m\pi}{Np} \quad (2)$$

Here,  $\beta_m$  is the propagation constant of mode,  $m$  is the mode number and  $N$  is the number of unit cells. Well-known metamaterial CRLH modes are achieved when  $m=0, \pm 1, \dots, \pm N-1$ . Therefore,  $2N-1$  resonances (i.e.  $N-1$  positive-order resonance (POR),  $N-1$  negative-order resonance (NOR) and

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one zeroth-order resonance (ZOR)) are achieved from an  $N$ -cell CRLH resonator.

In addition to CRLH resonances, another resonance frequency occurs at the higher frequency in comparison with CRLH modes when  $m=N$ . In this case, the equation (2) becomes  $\beta_m = \pi/p$  and this resonance locates at the edge of the first positive-order bandgap of the Brillouin zone. It should be noted that this resonance frequency is due to the unit cell resonating. Therefore,  $2N$  resonance frequencies are accessible from an  $N$ -cell CRLH resonator ( $2N-1$  CRLH resonances and one resonance of the unit cell). Mushroom structure as the simplest CRLH configuration which comprises a patch and via is examined here to investigate the theory. It is well-known that a simple rectangular patch antenna, at its fundamental mode, has two radiating edge with in-phase and two non-radiating edge with out-of-phase field distributions. By converting a simple patch to a mushroom resonator with via-process, it will also have CRLH features. Due to this conversion, CRLH resonances ( $2N-1$ ) are accessible according to the number of cells while the fundamental  $TM_{10}$  mode of the patch is preserved. This is beneficial especially in one-unit cell case due to dual-band operation.

In order to investigate the aforementioned theory, three microstrip antennas based on mushroom resonators with three-, two- and one-cell structure have been designed, simulated and fabricated.

### III. ANTENNA DESIGN

In this section to extract dispersion diagram, two different methods are used. One is using two-port simulation and employing scattering parameters, and the other is based on computing lumped-element values in equivalent circuit model. Figures 1(a) and (b) demonstrate schematic and dimensions of a mushroom unit cell with its equivalent circuit model. The structure dimensions are also labeled at the figure caption. Two-port simulation setup is adopted as shown in Fig. 1(a) and 1-D dispersion diagram is extracted. Two extra  $50\Omega$  microstrip sections are added to both ends of the unit-cell. It is imperative to de-embed the ports to eliminate phase shift due to the additional length of microstrip lines [9]. 0.2mm gap in the unit-cell refers to the gap between cells introducing the left-hand capacitance. With the evaluated scattering parameters, dispersion diagram is calculated by (with period  $p$ ):

$$\beta p = \cos^{-1} \left[ \frac{1 - S_{11}S_{22} + S_{12}S_{21}}{2S_{21}} \right] \quad (3)$$

The series parameters of the proposed structure are  $L_R=13.7$  nH and  $C_L=0.18$  pF. Resonance frequency of the zeroth-order mode due to open-ended termination is  $f_r = 1/(2\pi\sqrt{L_L C_R})$ . With the shunt parameters of  $C_R=1.97$  pF and  $L_L=1.58$  nH, the zeroth-order resonance frequency occurs at 2.85GHz.

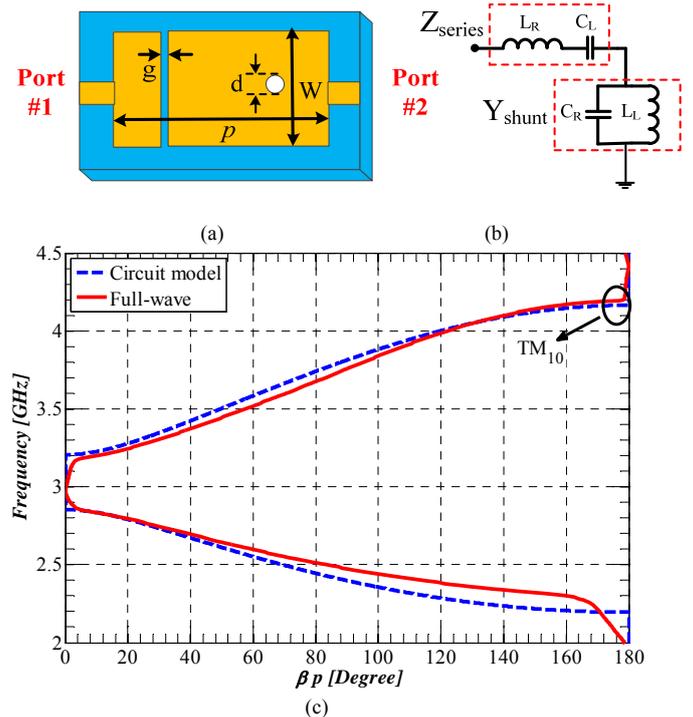


Fig. 1. (a) Two port simulation setup for mushroom structure on 1.524mm thick Rogers RO-4003 substrate with dielectric constant of 3.55,  $p=20.2$ mm,  $W=10$ mm,  $g=0.2$ mm,  $d=2.4$ mm, (b) equivalent circuit model with right- and left-hand parameters (c) dispersion diagram extracted by circuit model and full-wave simulation.

According to the equivalent circuit of Fig. 1(b), dispersion diagram is calculated using [Eq. 2, 9]. Figure 1(c) shows the dispersion diagrams of the proposed structure while good agreement between the two methods has been achieved. Based on aforementioned theory, it is expected that the  $TM_{10}$  mode will be located at the upper edge of the first RH band. According to Fig. 1(c), this mode happens at about 4.2GHz.

It should be noted that, symmetrical mushroom structure (via at the center of the patch) is investigated here. However, displacement of via from the center of the patch elaborates field distribution. Consequently, it provides asymmetrical field distribution at both zeroth-order and  $TM_{10}$  modes which may lead to shift both these resonance frequencies.

### IV. ANTENNA REALIZATION

In this section, three open-ended mushroom resonant antennas with different number of cells ( $N=3, 2, 1$ ) are investigated. The proposed antennas are implemented on 1.524mm thick Rogers RO-4003 substrate with dielectric constant of 3.55. A  $50\Omega$  proximity-coupled microstrip line is used as the feed network so that the input impedance is matched to  $50\Omega$  SMA connector. The proposed mushroom resonant antenna consisting of three periodically cascaded unit cells is illustrated in Fig. 2. Measured and simulated reflection coefficients of the proposed 3-cell CRLH mushroom resonant antenna and its manufactured

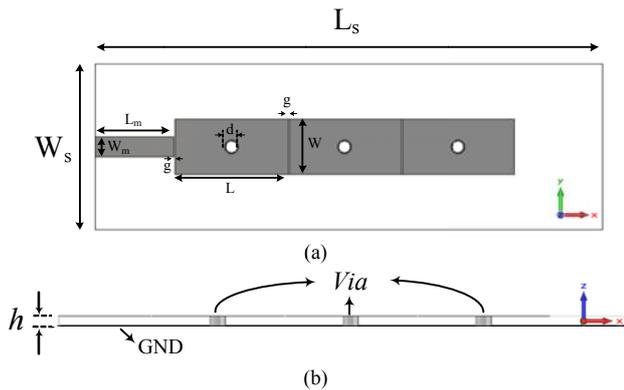


Fig. 2. Schematic of the proposed 3-cell CRLH mushroom resonant antenna (a) top view, (b) side view.  $L_s=90$ ,  $W_s=30$ ,  $L_m=13.8$ ,  $W_m=3.5$ ,  $L=20$ ,  $W=10$ ,  $d=2.4$ ,  $g=0.2$ ,  $h=1.524$  (dimensions are in mm).

prototype are shown in Fig. 3(a). The CRLH theory predicts that this 3-unit-cell antenna have five resonance frequencies. However, as illustrated in Fig. 3(a), there is an additional resonance frequency which belongs to  $TM_{10}$  mode. According to the dimension of the simple patch it is expected that the  $TM_{10}$  mode occurs at 3.83GHz. However, when identical patches are located besides each other, closer than  $0.2\lambda$  ( $\lambda$  is a wavelength), the resonance frequency is shifted to upper frequencies, which is due to coupling capacitors between identical patches [8]. The maximum frequency shift is observed at the dispersion diagram owing to assumption of infinite number of cells and periodicity condition. In the proposed 3-cell mushroom resonant antenna, the  $TM_{10}$  mode is shifted to 4.19GHz which is approximately matched with  $TM_{10}$  mode extracted from dispersion diagram in Fig. 1.

Reducing number of cells affects the number of resonance frequencies as expected by CRLH properties. Every time eliminating one unit cell in a CRLH mushroom resonator, one resonance from RH range and one from LH range are eliminated while the  $TM_{10}$  mode is preserved with a little frequency shift toward the fundamental resonance frequency of the patch (3.83GHz). Manufactured prototype of two-cell and single-cell mushroom resonant antennas with simulated and measured reflection coefficients are illustrated in Figs. 3(b) and (c). Two-cell mushroom resonant antenna provides three CRLH resonance frequencies plus  $TM_{10}$  mode at 4.1GHz. Finally, two resonance frequencies are achieved in a single-cell mushroom resonant antenna. Measured zeroth-order and  $TM_{10}$  modes are appeared at 2.88GHz and 3.88GHz, respectively. Slight frequency shift between simulated and measured results at some resonances may be contributed by fabrication process and dielectric constant tolerance in a frequency regime. Although, the  $TM_{10}$  mode occurs at a frequency above NOR, POR and ZOR frequencies and does not satisfy metamaterial cell size criteria, it is useful especially in one-unit cell mushroom resonator due to dual-band operation.

Simulated impedance diagram of the proposed single-cell antenna is compared with a simple rectangular patch with

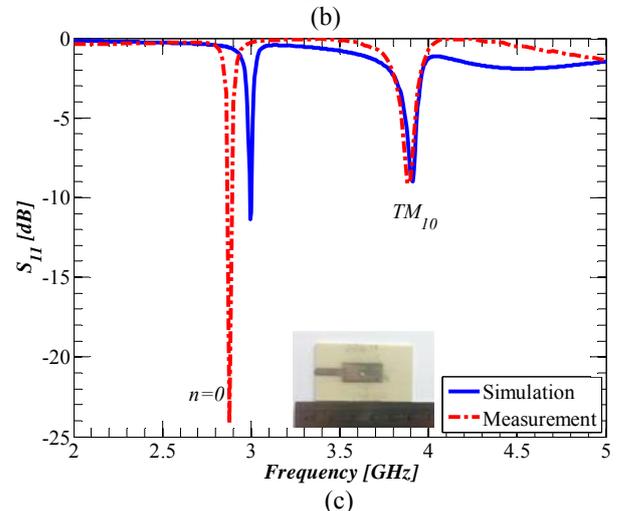
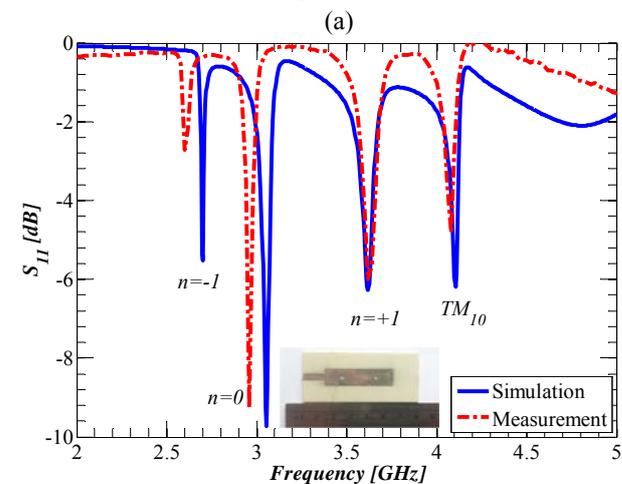
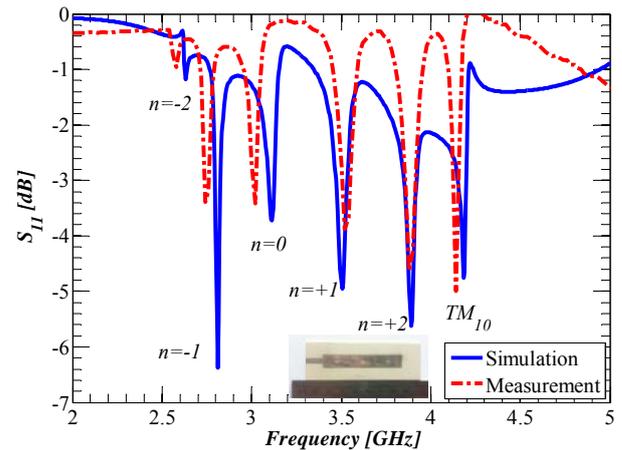


Fig. 3. Measured and simulated reflection coefficients of the proposed mushroom resonant antenna with its manufactured prototype, (a) 3-cell, (b) 2-cell, (c) 1-cell.

the same dimensions ( $L=20$ mm,  $W=10$ mm) in Fig. 4. It is noticed that a little frequency shift has occurred in  $TM_{10}$  mode and the zeroth-order mode is added in a single-cell mushroom resonant antenna. Figure 5 demonstrates simulated tangential electric field distribution at both zeroth-order and  $TM_{10}$  modes of the single-cell mushroom

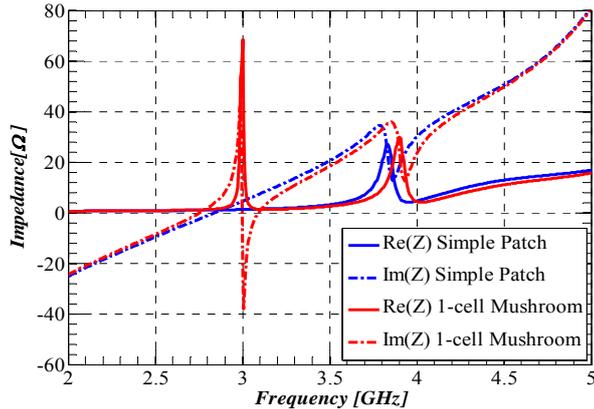


Fig. 4. Simulated input impedance of the proposed single-cell mushroom resonant antenna and the simple rectangular patch, that is the mushroom resonant antenna without via.

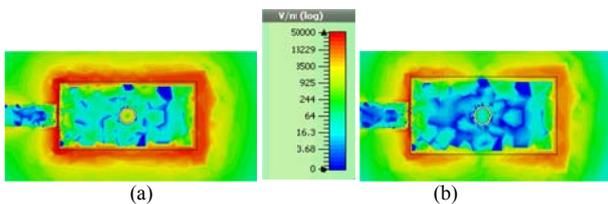


Fig. 5. Simulated tangential electric-field distribution of the proposed single-cell mushroom resonant antenna at (a) zeroth-order mode, (b)  $TM_{10}$  mode.

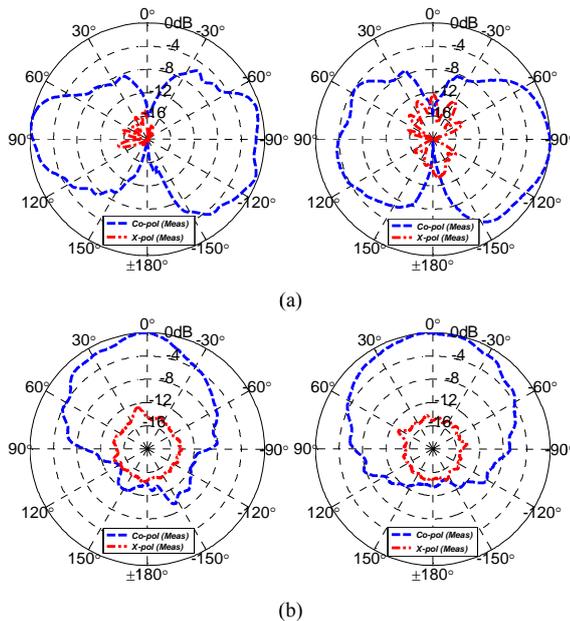


Fig. 6. Measured normalized radiation patterns of the proposed single-cell mushroom resonant antenna (a) zeroth-order mode, (b)  $TM_{10}$  mode. Left: XZ-plane, Right: YZ-plane.

resonant antenna. As mentioned in [10], there are in-phase electric field distribution along the whole perimeter of a unit-cell at zeroth-order mode in Fig. 5(a). However, at  $TM_{10}$  mode there are two radiating edges with in-phase and two non-radiating edges with out-of-phase field distributions (Fig. 5(b)).

The measured results of the normalized radiation patterns of the proposed single-cell CRLH mushroom resonant antenna at both zeroth-order and  $TM_{10}$  modes are shown in

Fig. 6. Due to in-phase field distribution at the zeroth-order mode, which is modeled as an equivalent magnetic loop, the monopolar radiation pattern is achieved (Fig. 6(a)). However, two equivalent magnetic current densities at the radiating edges of  $TM_{10}$  mode contribute to the common radiation pattern of a simple microstrip patch antenna (Fig. 6(b)). It should be noted that the unit-cell dimension is approximately  $0.26\lambda \times 0.13\lambda \times 0.019\lambda$ , at the  $TM_{10}$  mode ( $f_{TM_{10}}=3.88\text{GHz}$ ). Measured gain and simulated efficiency of the proposed antenna at this mode are 4.96 dBi and 78%, respectively. Moreover, the unit-cell dimension at the zeroth-order mode ( $f_0=2.88\text{GHz}$ ) is reduced to  $0.19\lambda \times 0.09\lambda \times 0.014\lambda$ . Measured gain and simulated efficiency at this mode are -0.82 dBi and 46%, respectively.

## V. CONCLUSION

In this paper, the zeroth-order and  $TM_{10}$  modes of the CRLH mushroom resonator have been investigated with regards to the number of cells. It has been proved that in addition to conventional  $2N-1$  resonance frequencies in an  $N$ -cell mushroom resonator, there is also a resonance. This extra resonance belongs to  $TM_{10}$  mode of the unit-cell and it is determined by the length of radiating edges. To verify this theory, three types of mushroom resonant antennas with different number of cells ( $N=3, 2, 1$ ) have been designed and fabricated. Finally, based on theoretical investigation, a single-cell dual-band CRLH mushroom resonant antenna has been proposed.

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