

Kinematic and Quasi-Kinematic Constraints: What They Are & How They Work

December 11, 2006

David Fellowes

Abstract:

Kinematic constraints allow a body to be held with the highest precision, exactly constraining each of the six degrees of freedom. Quasi-kinematic constraints allow a small amount of over-constraint while providing high precision. This tutorial will discuss both types of constraints to include detailed descriptions, advantages, disadvantages, and examples of each. The kinematic examples will be readily available COTS (commercial off the shelf) items, and the quasi-kinematic example will be a filter/reticle holder than can be readily made with a CNC mill.

Introduction:

Any object in three dimensional space can be defined with six independent coordinates: x, y, and z (three translation), and yaw, pitch, and roll (three rotation), as is shown in Figure 1 from Vukobratovich¹. When each of these degrees of freedom is constrained fully and very strictly none are over-constrained, the system is considered to be kinematically constrained. The theory of kinematic design requires perfectly rigid bodies touch only at points (point contacts). When slight over-constraints are allowed, the constraint is considered quasi-kinematic. See Figure 2 from Yoder² for examples of each.

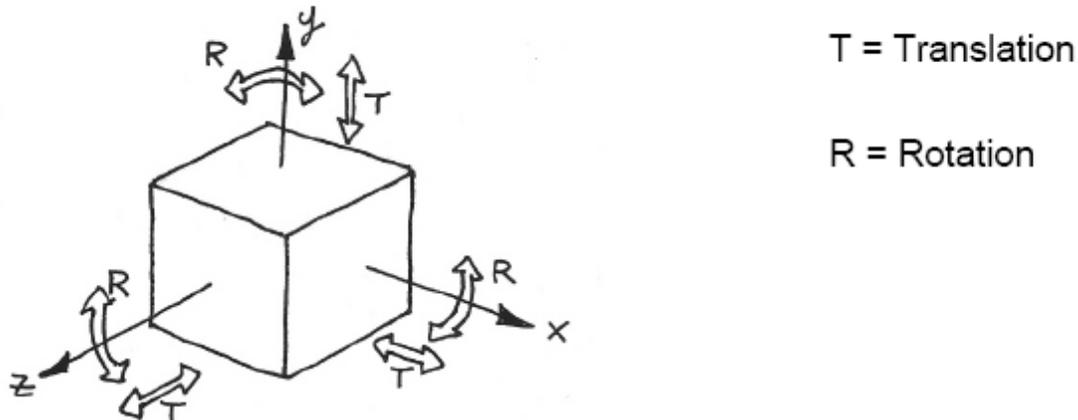


Figure 1. Six degrees of freedom: 3 in translation and 3 in rotation.

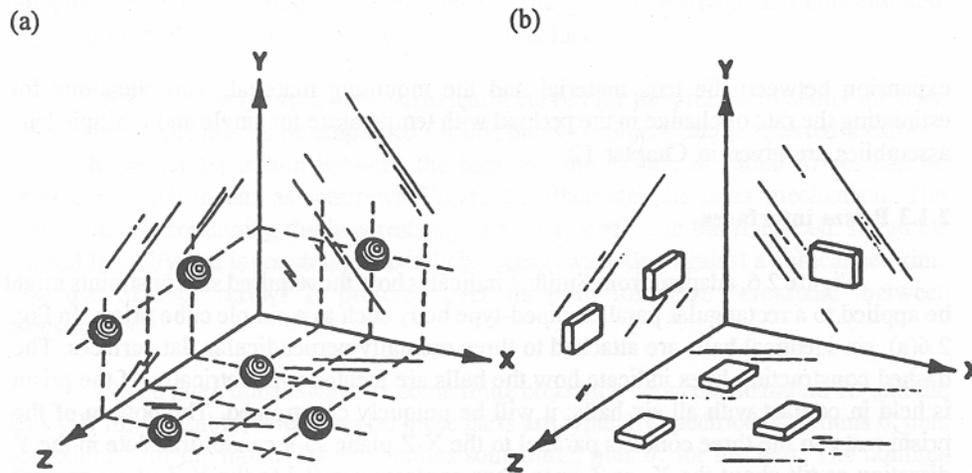


Fig. 2.6 (a) Kinematic and (b) semikinematic position-defining registration surfaces intended for interfacing with a cube-shaped prism (not shown). (Adapted from Smith.³)

Figure 2. A side-by-side comparison of a kinematic and quasi (or semi) kinematic mount for a cube. The kinematic mount's spheres provides point contacts while the quasi-kinematic mount provides area contacts (overconstraint).

Kinematic Constraints:

Description:

The kinematic constraint requires six point contacts that constrain all six degrees of freedom. (Note that it is possible to have six (or more) point contacts that do not constrain all six degrees of freedom.) A point contact is an idealized concept (and not really possible since all objects deform with pressure), but these contacts are considered points in conceptualizing the geometry, though they are not assumed to be point contacts in calculating the stresses. A hard sphere makes a good point contact when it meets another hard surface, which is why they are typically used in kinematic constraints.

Figure 2's kinematic mount representation is an excellent reference for seeing how each degree of freedom is constrained.

Looking at the bottom:

1 sphere constrains one translation direction. An object can pivot about this point in all directions and slide along this point in two directions.

2 spheres constrain one translation and one rotation.

3 spheres constrain one translation and two rotation directions. A plane has been defined that the object must stay in.

Looking at the left wall:

1 sphere (4 total now) constrains one translation. The object may still rotate in plane and shift in one direction.

2 spheres (5 total now) constrain on translation and one rotation. The object may only shift in one direction.

Looking at the back wall:

1 sphere (6 total now) constrains the final translation.

Whenever there is a degree of freedom that is not constrained, the system is under-constrained. This obviously happens when there are less than 6 contact points, but it can still occur with 6 or more points. In a system with 3 contacts in a plane and in a straight line, these 3 contacts will not adequately constrain the system in this plane; rocking can occur about the line of contact.

This system with three contacts in a straight line may also suffer from distortion due to over-constraint of one of the degrees of freedom. Whenever any degree of freedom is over-constrained, which is easy to see when more than 6 contacts are present, the system is not kinematically supported. Another example of an over-constrained system is one with 4 contacts in a plane (only 3 points are needed to define a plane); rocking and/or distortion will occur.

Preloading, which can be achieved with springs and/or gravity is an important concept for kinematic mounts. The preload is not a constraint, but it allows the constraint to be functional up to the amount of the preload in the opposite direction. The preload keeps the constraint components (such as a sphere and a flat) in contact. To avoid deformation and unwanted motion, the preload should be in line with the constraint.

Figure 2 shows a good way to kinematically constrain a cube prism, but other configurations are readily available from commercial vendors for generic kinematic mounting and positioning of optical systems. Such mounts typically feature three balls mating with either a cone, a V channel, and a flat or three V channels, as shown in Figure 3 from Burge³.

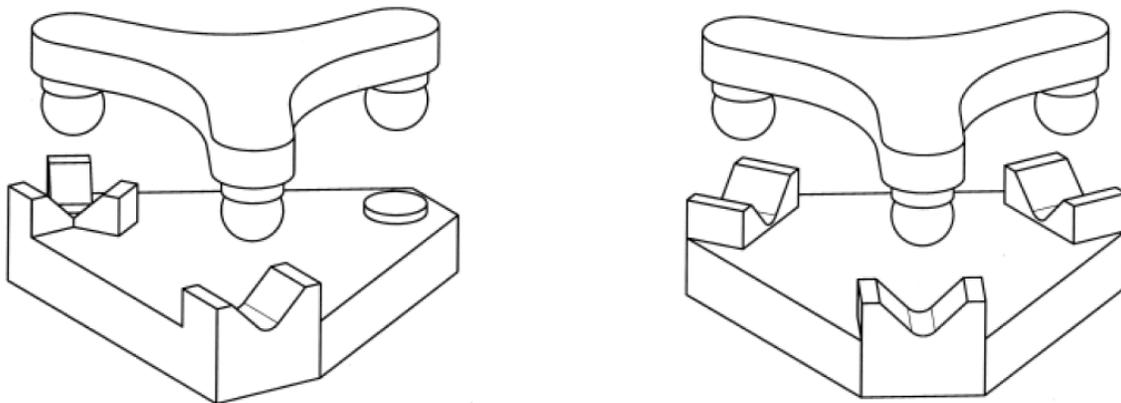


Figure 3. Two typical kinematic mounts from Burge. The one on the left has three spheres mating with a cone (simplified to 3 planes), a V channel, and a flat. The one on the right has three spheres mating with 3 V channels.

The kinematic mount with three balls, one cone, one groove, and one flat has exactly six point contacts. The ball-cone interface has three point contacts, effectively constraining all 3 translations. The ball-groove contact has two point contacts, constraining roll and

yaw. The ball-flat contact has one point contact, constraining pitch. Figure 4 from Burge shows each interface. Ideally each interface would be at the vertex of an equilateral triangle for good distribution of forces. An equilateral triangle is not necessary; however, the constraining forces should be designed to not be parallel and overlapping, which could leave the mount unstable or under-constrained, much like the example of 3 balls in line in a plane.

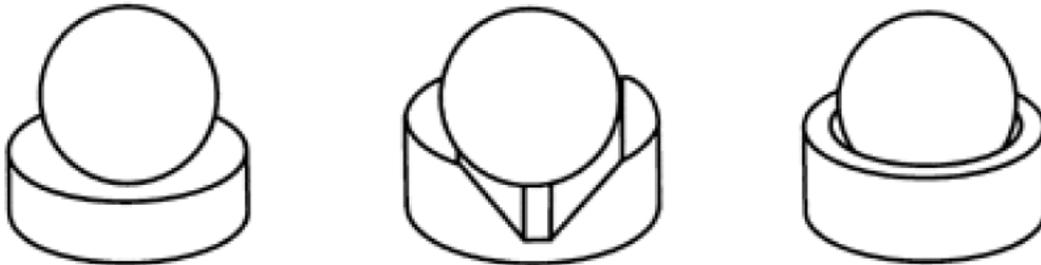


Figure 4. The three interfaces of the flat-groove-cone type kinematic mount. The flat provides only 1 constraint, the groove: 2, and the cone: the remaining 3.

The 6-point 3-groove kinematic system shown on the right side of Figure 3 is also a commonly available mount. Each interface provides 2 constraints, totaling 6 constraints for the system. The best stability is achieved when the normals to the contact planes bisect the coupling triangle (with each interface as a vertex of this triangle). This is shown in A from Figure 5 from Vukobratovich. The key is to have 6 point contacts that do not have forces overlapping or all meeting at a point. Figure 5 B meets this, though it is less stable than A. Figure 5 C has all forces meeting at the center of the triangle, so this mount would give poor stability. 3 parallel grooves (not shown) would not provide constraint in one translation direction.

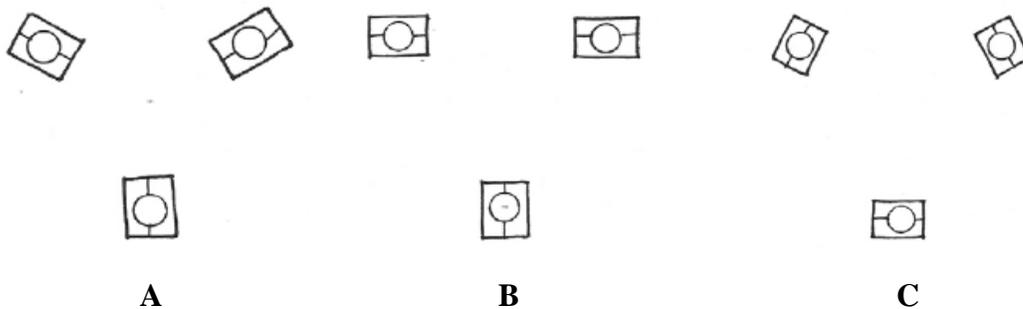
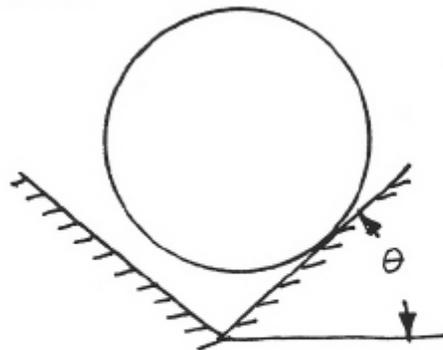


Figure 5. Stability with a 6-point 3-groove kinematic mount. A is the most stable, B provides some stability, and C provides poor stability.

The three fundamental concerns with all kinematic mounts are friction effects, contact stresses, and stability. The stability is handled with the geometry as discussed above for two different types. Materials that can handle stresses are essential, and efforts are made

to reduce these stresses where possible. Harder materials are often chosen for the contacts, but the reduced deformation (good) does result in higher stress.

The friction criteria for sliding must be satisfied with regards to the contact angles between kinematic loading points and locating features. This means it must be allowed to slide. If it cannot slide, the locating points will not set properly. From Vukobratovich, the coefficient of friction must be lower than the tangent of the angle (see Figure 6). If at all reasonable, sliding contacts should be replaced with rolling contact, though this increases the complexity and cost.



$$\mu \leq \tan \theta$$

Figure 6. The requirement of the coefficient of friction for kinematic contacts.

Advantages & Disadvantages:

Kinematic systems and the use of kinematic coupling provide many benefits. These include holding a body with the highest precision, motion without any backlash or play, and repeatable (to the sub-micron level) removal and replacement of a part to the same location. These features are often important for optical alignment when such precision is essential. Another benefit is that kinematic mounts can be built without precision manufacturing techniques.

Kinematic constraints are not without problems or disadvantages. Any overconstraint would cause problems: either lurch or deformation. Friction, surface finish, and loads can make true kinematic constraint very difficult and ruin repeatability. Point contacts are not truly possible (making kinematic constraints not truly possible) since all materials deform with force, and greater force causes greater deformation. When very stiff materials are used to minimize deformation, the stress can get very high since it is the force divided by the surface area. Since preloads are required to maintain constraint, stresses increase and the tangential effects due to friction may be large.

Culpepper⁴ also discusses how kinematic coupling fails to achieve 3 low cost manufacturing goals. 1. Low-cost generation of fine surface finish: inexpensive fine surface finish balls are readily available, but fine surface finish grooves are not. 2. Low-

cost generation of alignment feature shape: balls and grooves are more complex than pin joints, and the surface hardness processing required that allow joints to survive Hertzian contact stresses drives the costs higher. 3. Low-cost means to form sealed interfaces: adding sealing flexures to kinematic couplings increases costs.

Example:

An excellent example of a kinematic system, a Newport Corporation optic mount, is shown in Figure 7. The mount utilizes three stainless steel balls, a flat, a groove, and a cone to achieve the kinematic constraints. Two steel balls are at the end of actuators that provide adjustment (pitch and yaw). One of these balls rests on a flat, making one “point” contact, controlling one DOF. The other actuator ball rests in a groove, making 2 “point” contacts, controlling two DOF’s. The third ball sits at the pivot point, resting in a cone, making 3 “point” contacts, controlling the remaining three DOF’s. The two restraining forces, provided by springs, are located close to each actuator. The close proximity and use of springs helps to avoid overconstraint of the system. It should be noted that as adjustments are made with the actuators, the optic will move in pitch and yaw (as expected), and it will experience some translation in x, y, and z since the center of rotation (the pivot point at the third ball) is not at the center of the optic, as it is in gimbaled systems.



Figure 7. Newport adjustable kinematic optic mount.

Quasi-Kinematic Constraints:

Description:

Quasi-kinematic (or semi-kinematic) constraints are similar to kinematic constraints, but they allow slight overconstraints. Some (or all) point contacts are replaced with area contacts. See Figure 8 for sketches of quasi-kinematic constraints.

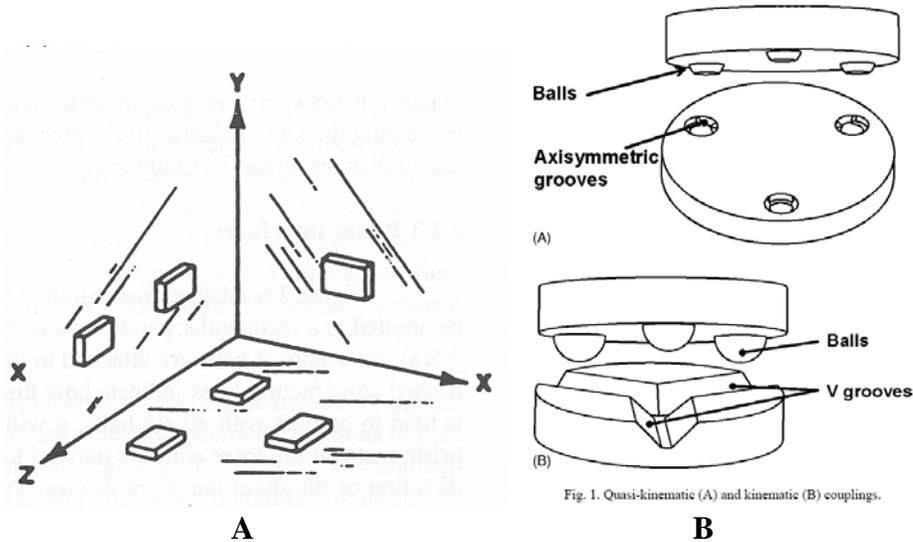


Fig. 1. Quasi-kinematic (A) and kinematic (B) couplings.

Figure 8. Quasi-kinematic constraints. A. Shows a cube optic mount from Figure 2 (Yoder). The balls have been replaced with small flat surface. **B.** Culpepper shows a quasi-kinematic variation of the 6-contact 3-groove mount where the three grooves are replaced with arc-grooves, forming a surface contact.

Advantages & Disadvantages:

Quasi-kinematic mounts provide good repeatability (10's of microns) with less cost than kinematic mounts. Stresses are significantly lower, and the mounts can be readily machined on a computer numerically controlled mill.

Culpepper discussed how his design of quasi-kinematic mounts improves on the three low-cost coupling manufacturing goals. 1. Low-cost generation of fine surface finish: High quality grooves can be achieved by burnishing the groove's surfaces by pressing the harder, finer ball into it. 2. Low-cost generation of alignment feature shape: the QKC groove can be made in simple drill operations using countersinks or form tools since the grooves are axisymmetric. Groove reliefs can be made in place by drilling, forming, milling, or casting with comparable costs to pinned joints. 3. Low-cost means to form sealed interfaces: by making the ball contact feature hollow, by adding an undercut, and by providing a sufficient nesting force, the gap between the two components may be closed if the ball-groove materials plastically deform during the first mate. Elastic recovery will allow a portion of the gap to return, which is necessary in maintaining its kinematic nature.

Example:

The semi-kinematic mount shown in Figure 9 can be used to hold a filter or reticle kinematically. This filter holder with semi-kinematic features can be manufactured on a CNC mill and thus delivered quickly. The filter sits on three contact pads, providing constraint in 3 DOF's (z, pitch, and yaw). The springs, which provide the restraining force, are located close to the pads to prevent warping and additional overconstraint (the hand sketch and the Pro/E drawing are not identical). Two protrusions on one edge limit two more DOF's (y and roll). A protrusion on the other edge limits the remaining DOF

(x). This mount can hold a filter precisely without warping it excessively or damaging it while allowing light to pass through the middle of the aperture in the holder.

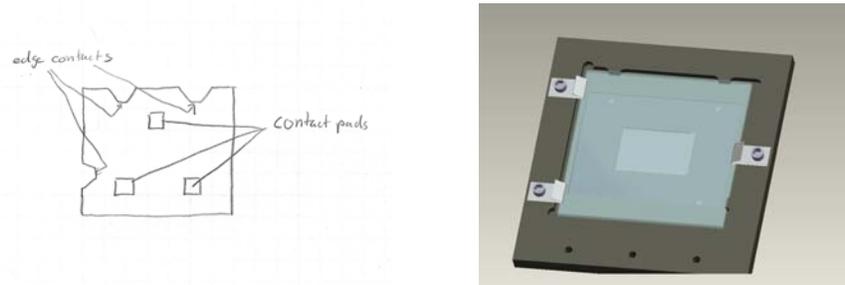


Figure 9. Quasi-kinematic filter holder.

Conclusion:

Kinematic coupling allows the precise constraining of any object in all 6 degrees of freedom. If all six degrees of freedom are constrained without over-constraint, the system can be considered to be kinematically constrained. Although a loss of performance results, quasi-kinematic constraints, which allow slight over-constraints, provide good precision while being much less expensive. Both types of mounts are used extensively in optomechanical design and are important concepts to remember.

References:

1. D. Vukobratovich, S. Vukobratovich, "Introduction to Opto-Mechanical Design", University of Arizona Class Notes, 66-80 (2006).
2. P. Yoder, "Mounting Optics in Optical Instruments", SPIE – The International Society for Optical Engineering, Bellingham, WA, 26-29 (2004).
3. Burge, "Principals of Kinematic Constraint", University of Arizona Class Notes for Introductory to Opto-Mechanical Engineering (2006).
4. M. L. Culpepper, "Design of quasi-kinematic couplings," Precision Engineering 28 338–357 (2004).