PARTIAL POLAROIDS IN BIREFRINGENT FILTERS

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Abstract. It is demonstrated that a single partial polaroid in a Lyot filter behaves in much the same manner as a contrast element. Use of a partial polaroid with a transmission ratio of 10 to 1 results in a factor of 10 decrease in the principal secondary maxima. An explanation of the effect of the partial polaroids is presented in terms of the pulse response of the birefringent network.

1. Introduction

Birefringent filters have been extensively discussed in literature. There are even papers discussing how birefringent elements can be used in combination to produce arbitrary band shapes. However, except for the paper of Giovanelli and Jefferies (1954) which showed that a Lyot filter constructed with partial polarizers had less transmission in the unwanted secondary maxima and the verification of their calculation by Dunn and Beckers (1965), little work has been done on the effect of partial polarizers in birefringent stacks.

In this paper, the physical effect of partial polaroids in normal Lyot configurations will be described. It will be shown that proper use of partial polarizers can suppress the secondary maxima of a standard Lyot configuration.

Further, it will be shown that the action of partial polarizers is very similar to that of the standard contrast element technique for suppression of the Lyot side bands. The method of pulse response discussed by Mertz (1965) and by Harris *et al.* (1964) for describing the action of birefringent stacks will be used to establish the frequency response of the filters. The general problem of birefringent networks with partial polarizers will be analyzed in a later paper.

2. Effect of a Partial Polaroid

Before discussing the properties of filters in terms of their pulse response, it is useful to understand partial polarizers in terms of an interference phenomenon. Shown in Figure 1 is a simple 4-element Lyot filter configuration in which the fast axes of adjoining crystal elements are orthogonal, and the crystals are arranged in order of decreasing size. The length of each crystal is twice that of the element following it.

Although in a normal Lyot filter the crystals may be $\pm 45°$ to the polarizer transmission axis and the order of the crystals is unimportant (Evans, 1949), both the orientation of the crystals and their order is critical in a filter constructed with partial polarizers.

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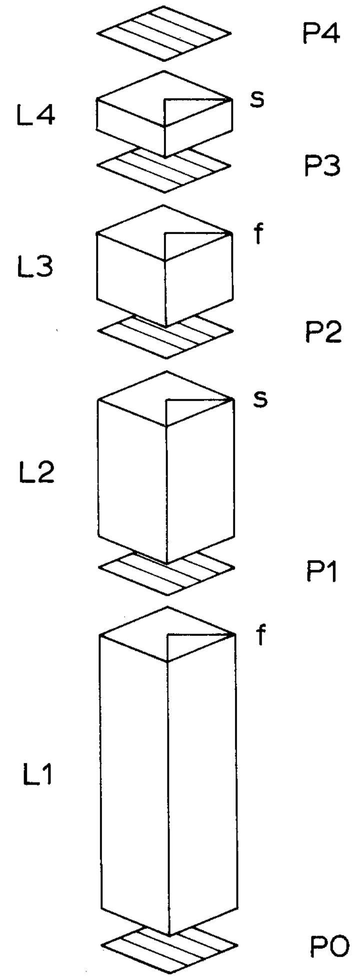

Fig. 1. Optical schematic of 4-element Lyot filter with alternate elements orthogonal.

If the longest crystal has an optical thickness difference L_1, then to an excellent approximation, the electric field transmitted in the neighborhood of a transmission maximum is

$$E(\Delta\lambda) = \sin\left(\frac{\pi}{\varphi}\Delta\lambda\right) \Big/ \left(\frac{\pi\Delta\lambda}{\varphi}\right) \tag{1}$$

or

$$E(\Delta\lambda) = \operatorname{sinc}\left(\frac{\pi}{\varphi}\Delta\lambda\right), \tag{2}$$

where

$$\varphi = \frac{2\lambda_0^2}{L_1}, \tag{3}$$

$$L_1 = (n_0 - n_e)\, l_1 , \tag{4}$$

$$\Delta\lambda = \lambda_0 - \lambda , \tag{5}$$

where l_1 is the length of the longest element, n_o is the ordinary index, n_e is the extraordinary index, λ_o is a wavelength of maximum transmission, and λ is the operating wavelength.

The transmitted intensity is proportional to the square of the electric field strength. Shown in Figure 2, curve *a*, is a plot of the emergent electric field strength near a transmission maximum for the filter of Figure 1.

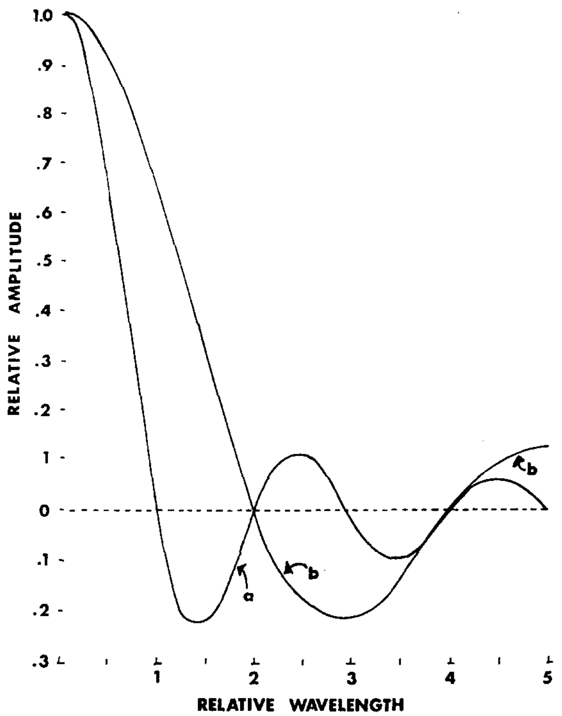

Fig. 2. Transmitted electric field intensity for a Lyot filter of length L (a) and $L/2$ (b).

Now suppose the polarizer, P_1, between the first and second elements of the filter is removed. Since the fast axes of the two adjoining elements are crossed, their birefringence subtracts. Thus the two elements act in combination like a single element of length $L_1/2$. Since each element is twice the length of the following element, the new combined 'first element' has the same optical thickness as the second (original) element. Hence, with the polaroid removed, the effective length of each 'element' is still twice that of the following element, and the fast axes are still at ± 45 deg to the polaroid axis. Therefore, the electric field strength transmitted with the polaroid removed will be that of the Lyot filter with the largest element removed. That is,

$$E(\Delta\lambda) = \operatorname{sinc}\left(\frac{\pi}{2}\frac{\Delta\lambda}{\varphi}\right). \tag{6}$$

Shown in Figure 2, curve b, is the electric field amplitude for the filter with P_1 removed. Comparison of curves a and b illustrates that the electric field amplitudes of the two configurations have opposite signs in the region of the first two amplitude extremals of curve a.

If a filter could be made as a linear combination of a perfect polaroid filter and a filter with P_1 removed, the first two secondary maxima would be considerably reduced. A partial polaroid creates just such a linear combination and the adding of electric field amplitudes is a simple interference effect. Mathematically a partial polaroid can be described by a Jones (1941) matrix of the form

$$PP = \frac{1}{\sqrt{2}}\begin{pmatrix} \varrho_1 & 0 \\ 0 & \varrho_2 \end{pmatrix}, \tag{7}$$

where the transmittances in the x and y directions are given by

$$T_x = \varrho_1^2/2, \tag{8}$$

$$T_y = \varrho_2^2/2. \tag{9}$$

The matrix can also be written in the form

$$PP = \frac{(\varrho_1 - \varrho_2)}{\sqrt{2}}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + \frac{\varrho_2}{\sqrt{2}}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{10}$$

The first term in Equation (10) represents a neutral density filter plus a perfect polaroid aligned along the x axis, while the second term represents a pure neutral density filter. Thus if the perfect polarizer, P_1, is replaced by a partial polarizer the resulting filter behaves as a linear combination of Lyot filters of lengths L_1 and L_2. With a partial polarizer at the location of P_1, the electric field is

$$E(\Delta\lambda) = \frac{1}{\sqrt{2}}\left\{(\varrho_1 - \varrho_2)\operatorname{sinc}(\pi\Delta\lambda/\varphi) + \varrho_2 \operatorname{sinc}\left(\frac{\pi\Delta\lambda}{2\varphi}\right)\right\} \tag{11}$$

or

$$E(\Delta\lambda) = \frac{\varrho_1 - \varrho_2}{\sqrt{2}}\left\{\operatorname{sinc}(\pi\Delta\lambda/\varphi) + \varepsilon \operatorname{sinc}\left(\frac{\pi\Delta\lambda}{2\varphi}\right)\right\}, \tag{12}$$

where

$$\varepsilon = \varrho_2/(\varrho_1 - \varrho_2). \tag{13}$$

Shown in Figure 3b is the transmitted intensity for $\varepsilon = \frac{1}{2}$ and in 3a for the perfect polarizer filter.

3. Pulse Response Description of Partial Polaroids and Contrast Elements

Normally the secondary maxima of Lyott filters are suppressed with one or more contrast elements. Typically, these contrast elements are crystal elements of lengths shorter than the longest element of the filter (Schoolman, 1973). By means of pulse response analysis it is easy to show that the pulse response of a partial polariod filter and a contrast element filter are quite similar.

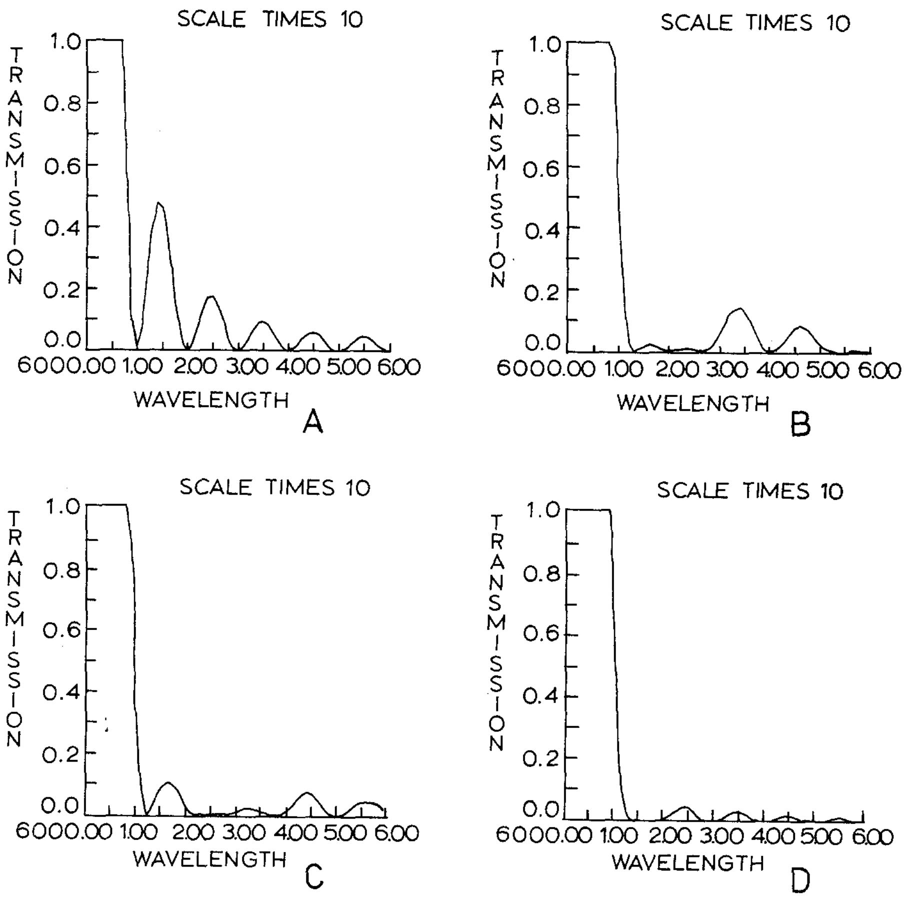

Fig. 3. Scale × 10 transmissions profile of normal Lyot filter (a), single partial polaroid filter (b), contrast element filter (c), two partial polaroid filter (d). All filters are of the same total length.

Although the pulse response of a Lyot system has been discussed in the literature (Harris *et al.*, 1964; Mertz, 1965) it is worthwhile to repeat it here since the explanation of partial polarization filter effect depends heavily on a solid understanding of pulse response of the standard network.

If a light pulse is incident on a single Lyot element with perfect polarizers on both ends two pulses will emerge separated in time by

$$\delta(l) = \frac{l}{c} \Delta n, \tag{14}$$

where Δn is index difference and c is the speed of light in vacuum.

For convenience the time origin can be redefined, so that the two pulses arrive at $\pm \delta/2$.

Shown in Figure 4a is a pulse 'tree' for a four element Lyot filter, where the initial pulse is incident on the longest crystal. The output of an n-element crystal is 2^n pulses

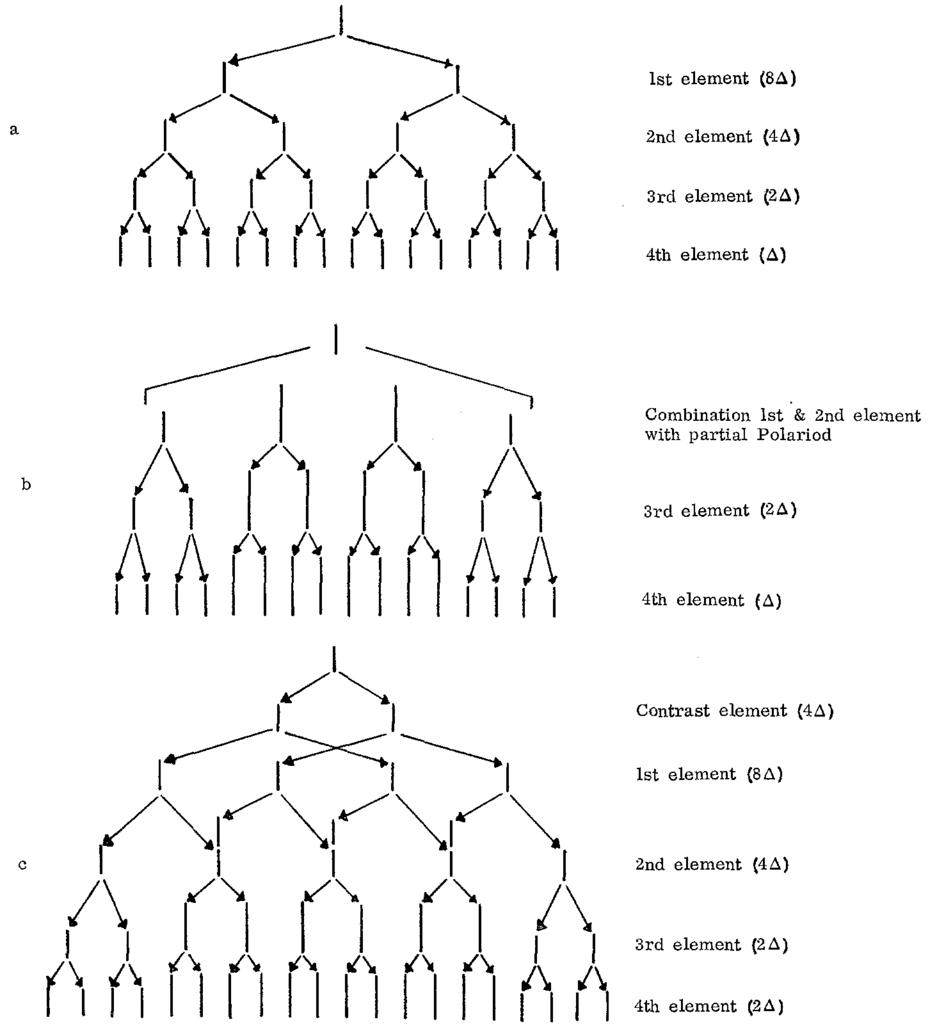

Fig. 4. Pulse response of 4-element Lyot filter (a), single partial polaroid filter (b), contrast element filter (c).

(Harris, 1964) all of which have the same amplitude. The separation of individual pulses is just the time separation due to the shortest element, and the separation between the first and last pulse is the sum of the time separation due to all of the elements As the number of elements approaches infinity the total time delay approaches twice the delay of the longest element.

Consider a partial polaroid inserted between the first and second elements. The output of the first two elements is four pulses, the inner two of which are of amplitude $\varrho_1+\varrho_2$ and outer two of which have amplitude $\varrho_1-\varrho_2$. If epsilon is greater than zero, the pulse amplitude distribution for the partial polarizer filter is hat shaped; such a distribution is shown in Figure 4b. If $\varepsilon=\frac{1}{2}$, the amplitude of the center eight pulses are twice the amplitude of the outer eight.

The relation between a partial polaroid system and a contrast element Lyot filter can now be easily demonstrated. A contrast element filter is simply a standard Lyot filter with an extra element whose length is typically equal to the length of the second largest element. Shown in Figure 4c is the pulse amplitude distribution for such a filter. Note that the pulse amplitude distribution from the contrast element filter is also hat shaped.

The inner pulses of the distribution are larger than the outer pulses because pulses overlap in the contrast element pulse tree. As seen from Figure 3c the overlapping occurs at the output state of the second element. Since pulses arriving at the same time are indistinguishable, two simultaneous pulses of equal amplitude are the same as a single pulse of double amplitude. As the number of elements of a Lyot filter with a contrast element filter approaches infinity the ratio of the number of double amplitude pulses to the total number of pulses approaches $\frac{3}{5}$.

Mertz (1965) and Harris (1964) showed that the fourier transform of the envelope of the pulse amplitude distribution is the electric field intensity as a function of frequency near a transmission maximum. Knowing that the field intensity for a uniform pulse distribution (the Lyot distribution) is the sinc function, it follows that the electric field of a contrast element filter and a partial polaroid filter ($\varepsilon=\frac{1}{2}$) near a peak are:

$$E_c(\Delta\lambda) = \tfrac{1}{2}\left\{\operatorname{sinc}\left(\pi\frac{\Delta\lambda}{\varphi}\right) + \tfrac{3}{5}\operatorname{sinc}\left(\frac{3\pi}{5}\frac{\Delta\lambda}{\varphi}\right)\right\}, \tag{15}$$

$$E_{PP}(\Delta\lambda) = \tfrac{1}{2}\left\{\operatorname{sinc}\left(\pi\frac{\Delta\lambda}{\varphi}\right) + \tfrac{1}{2}\operatorname{sinc}\left(\frac{\pi}{2}\frac{\Delta\lambda}{\varphi}\right)\right\}. \tag{16}$$

Shown in Figure 3b and c are plots of the transmission of the partial polaroid and contrast element filter made with the same total length of calcite.

4. Multiple Partial Polaroids

So far the situation for only a single partial polaroid has been discussed. In contrast, the calculations of Giovanelli and Jefferies (1954), and Beckers and Dunn (1965) were for filters with all polarizers partial. It can be shown that for the configuration of Figure 1, which is the one discussed by Giovanelli and Jefferies, a partial polarizer between the longest and next longest element aids in the suppression of the secondary maxima, while all the other partial polaroids enhance the secondary maxima.

From pulse response arguments, it can be shown that it is possible to add an additional partial polarizer that acts to suppress the secondary maxima. The two partial polarizer configurations requires that both the second and third longest elements adjoin the longest element with their fast axis orthogonal. Partial polarizers can be used on both ends of the longest element. Shown in Figure 3 d is transmission of such a filter. In a subsequent paper it will be shown that it is possible to construct a filter with half of the polarizers partial. However, such a filter is not a standard Lyot configuration.

5. Experimental

In order to verify the theoretical prediction for partial polaroid filters, a partial polaroid section was constructed by H. E. Ramsey of Lockheed Solar Observatory. This section consisted of two HN-38 end polaroids, an HN-55 center polaroid, and two birefringent elements with a length ratio of 2:1. The fast axes of the birefringent elements were orthogonal. For the test $\varepsilon = \frac{1}{2}$, since HN-55 transmits nearly 100% in the pass direction and 10% in the suppression direction (see Equations (8), (9) and (13)).

When the configuration was tested on the spectrograph, it did *not* perform as calculated. However, it was soon realized that the substrate material for all sheet Polaroid is birefringent with a nearly quarter-wave delay at 5200 Å (Title, 1973) between the transmission and suppression directions. A waveplate was made to cancel the retardation of the HN-55. When the section was reassembled it performed as calculated to within measurement errors.

6. Discussion

It is often stated that a contrast element narrow a filter. This is, of course, true since the FWHM of the total filter goes down by a factor of 0.926 if the contrast element added is half the length of the longest element. However, the contrast element increases the amount of calcite in the filter. If the length of the longest element in a contrast element filter is l, then the total amount of calcite is approximately 2.5 l. A normal non-contrast element filter has a total amount of calcite equal to twice the longest element. If a contrast element filter and non-contrast element filter are made of the same total length of calcite, then the length of the longest element in the contrast element filter is $\frac{4}{5}$ of the longest in the normal filter. This means that without the contrast element the contrast element filter has a FWHM of 1.25 that of the normal filter. Adding the contrast element reduces the width to (0.926×1.25) 1.158 that of the normal filter with the same amount of calcite.

Shown in Figures 3b and c are plots of the transmission of a partial polaroid filter ($\varepsilon = \frac{1}{2}$) and a contrast element filter of the same total length of calcite. Note that the position of the first zero and the suppression of the secondary maxima are similar.

The use of a partial polarizer filter, as compared to a contrast element filter, saves the construction of a contrast element. More importantly, it eliminates the extra polarizer of the contrast element and allows replacement of a high efficiency polarizer by one of low efficiency, i.e., high transmission. If the Polaroid data for HN-38, the normal good polaroid, and HN-55, the low efficiency polaroid, are used to compare the performance of partial polarizer and contrast element filters, it is found that a 40% transmission increase results from the use of the partial polarizer configuration.

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