

A concentrator for static magnetic field

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Abstract:

We propose a compact passive device as a super-concentrator to achieve an extremely high uniform static magnetic field over 50T in a large two-dimensional free space in the presence of a uniform weak background magnetic field. Our design is based on transformation optics and metamaterials for static magnetic fields. Finite element method (FEM) is utilized to verify its performance.

1. Introduction and background

In recent years, many novel optical devices have been designed and experimentally demonstrated with transformation optics (TO) [1-5]. TO is based on the invariance form of Maxwell's equations which can establish a one-to-one mapping between two spaces (a reference space and a physical space). By choosing transformation functions and reference space tactfully and creatively, one can design some optical devices with peculiar functionalities, such as invisible cloaks [6-8], perfect lenses [9, 10], concentrators [11, 12], field rotators [13], beam splitters [14], polarization controllers [15], optical black holes [16, 17], illusion optical devices [18], etc.

More and more attention has recently focused on controlling static fields by TO. Pendry et al have used TO to design plasmonic devices in quasi-static condition [19]. A DC electric cloak [20] and a DC electric concentrator [21] have been proposed and experimentally realized in the past year. In the development of DC magnetic metamaterials [22, 23], many DC magnetic devices based on TO (e.g., DC magnetic cloaking [24, 25] and DC magnetic energy concentrators [26]) have also been studied in recent years.

In fact, the static or quasi-static magnetic fields play an essential role in many areas, including metal detection [25], magnetic sensing [27], and wireless energy transmission [28]. Concentrating a DC magnetic field to obtain a higher magnetic field in free space has important applications for increasing the sensitivity of a magnetic sensor [29, 30], improving the performance of transcranial magnetic stimulation [31, 32], improving the spatial resolution of magnetic resonance imaging (MRI) [33], etc. High magnetic fields have been used in many areas, including physics, biology, materials science, and engineering. Very high magnetic fields are usually generated by resistive and/or superconducting magnet systems [34, 35]. However, energizing such magnets requires extremely high electrical power. So far the strongest artificial DC magnetic field is up to 45T at the National High Magnetic Field Laboratory in the United States, which consumes about 30 MW of electrical power [36].

In this paper, we will summarize how to use TO to enhance static magnetic fields and design a super-concentrator, which is a passive compact device and can realize a DC magnetic field larger than 50T inside it (free space region) when applying a uniform weak background magnetic field on it. We can also use the device proposed in ref. [26] to obtain a large DC magnetic field over 50T in 2D free space. However in order to obtain a static magnetic field larger than 50T, the ratio of outer radius to inner radius of the device in [26] should be larger than 50, which means we have

to sacrifice space to obtain an extremely high magnetic field. Based on folded transformation, our super-concentrator can achieve an extremely high magnetic field in a large free space with a very small ratio of outer radius to inner radius.

We will first review the TO for static magnetic field. Then we will summarize the different methods for enhancing the static magnetic field by using TO and then propose a super-concentrator for static magnetic fields. Finite element method is used to verify our ideas, and comparison with other methods is also given.

2. Transformation optics for static magnetic field

We will first summarize the TO for static magnetic fields. As the static field is the limiting case when $\omega \rightarrow 0$ in Maxwell's equation, TO for the static field can directly be derived from the general TO formula by setting $\omega \rightarrow 0$. In this case, the electric field and magnetic field are decoupled. We can separately design permittivity and permeability by coordinate transformation to control the static electric field and magnetic field. As Maxwell's equations are invariant under the coordinate transformation, divergence and curl equations for static magnetic field are also invariant under the coordinate transformation. The equations for the static magnetic field in a source free space can be written as:

$$\bar{\nabla} \times \bar{H} = 0, \quad \bar{\nabla} \cdot \bar{\mu} \bar{H} = 0 \quad (1)$$

The coordinate transformation can be expressed by using Jacobian transformation matrix M . if we want to keep the form of Eq. (1) invariant, the field and material should be transformed accordingly by:

$$\bar{\mu}' = M \bar{\mu} M^T / \det(M), \quad \bar{H}' = (M^T)^{-1} \bar{H} \quad (2)$$

We should note that quantities in physical space and in reference space are expressed with and without superscript separately in this paper.

3. Enhance static magnetic field by TO

We can give various ways to enhance DC magnetic field by using TO, for example we can simple compress a big volume to a small one while keeping outside an identical transformation to enhance the field (see Fig. 1 (a) and (b)), which has been proposed in [11, 12, 26]. Based on this configuration, if we want to obtain an extremely large DC magnetic field, we should extremely compress the space. It means that if we want to obtain a very high DC magnetic field in a small region, the size of the device based on this method should be very large (see Fig. 2 (a)). We can apply methods for designing optical black holes [16, 17] or some other non-Euclidean space based devices [8, 37-39] to design a magnetic black hole or a magnetic harvester for concentrating the DC magnetic field. However devices based on this method can only achieve high magnetic field in high permeability medium but not in free space by using magnetic black holes. Many applications need high magnetic field in free space but not high permeability medium [29-33].

However these methods can only concentrate the DC magnetic field in magnetic materials but not in free space. Inspired by superscatterers designed by TO for enlarging the scattering cross section [40, 41], we can also use space-folded transformation to concentrate DC magnetic field. By using this transformation, as we will show later, we can obtain a uniform DC magnetic field $B=50T$ or even larger in 2D free space within a very compact passive device (see Fig. 2(b)).

We consider cylindrical shell (infinitely long in the z direction) with inner radius R_1 and outer

radius R_2 respectively. The transformation can be written as:

$$\rho' = \begin{cases} \frac{R_1}{R_3} \rho, \rho' \in [0, R_1) \\ \frac{R_2 - R_1}{R_2 - R_3} \rho + R_2 \frac{R_1 - R_3}{R_2 - R_3}, \rho' \in [R_1, R_2]; \theta' = \theta; z' = z. \\ \rho, \rho' \in (R_2, \infty) \end{cases} \quad (3)$$

The transformation medium can be calculated:

$$\mu' = \begin{cases} \text{diag}(1, 1, \left(\frac{R_3}{R_1}\right)^2), \rho' \in [0, R_1) \\ \text{diag}\left(\frac{\rho' - R_2}{\rho'} \frac{R_1 - R_3}{R_2 - R_3}, \frac{\rho'}{\rho' - R_2} \frac{R_1 - R_3}{R_2 - R_3}, \left(\frac{R_2 - R_1}{R_2 - R_3}\right)^2 \frac{\rho' - R_2}{\rho'} \frac{R_1 - R_3}{R_2 - R_3}\right), \rho' \in [R_1, R_2] \\ 1, \rho' \in (R_2, \infty) \end{cases} \quad (4)$$

For a 2D case (magnetic field is in the plane), only μ_r and μ_θ effect the magnetic field. We can rewrite Eq. (4) as:

$$\mu' = \begin{cases} 1, \rho' \in [0, R_1) \\ \text{diag}\left(\frac{\rho' - R_2}{\rho'} \frac{R_1 - R_3}{R_2 - R_3}, \frac{\rho'}{\rho' - R_2} \frac{R_1 - R_3}{R_2 - R_3}\right), \rho' \in [R_1, R_2] \\ 1, \rho' \in (R_2, \infty) \end{cases} \quad (5)$$

As we can see the device is filled in the region $\rho' \in [R_1, R_2]$. Both inside and outside of the device is air in 2D case. For a traditional concentrator, it requires $R_1 < R_3 < R_2$ (see Fig. 1(a)). For the magnetic energy harvesting device recently proposed in [26], it still satisfies $R_1 < R_3 < R_2$ but the only difference is R_3 infinitely approaches R_2 (see Fig. 1(b)). That is also the reason why they obtain $\mu_\theta \rightarrow 0$ and $\mu_\rho \rightarrow \infty$. For a super-concentrator, we have $R_1 < R_2 < R_3$ (see Fig. 1(c)). In this case, the degree of concentration is not limited by the size of device. However we have to introduce negative anisotropic permeability to fulfilling the device.

We use finite element method (FEM) to verify our design. As shown in the Fig. 2(b), if we apply a uniform background static magnetic field whose magnetic flux density is 1T, the static magnetic field inside our device (it is free space) can achieve 50T which cannot be achieved by traditional methods [35, 36]. The material parameters of our device are shown in the Fig. 3. We should note that we can theoretically achieve higher magnetic field (much larger than 50T) by our method, which will require highly anisotropic negative permeability materials. As we have mentioned, if we use the magnetic harvesting device designed in ref. [26], we have to use a device whose geometrical size is extremely large to obtain a high magnetic field in a small region (see Fig. 2(a)).

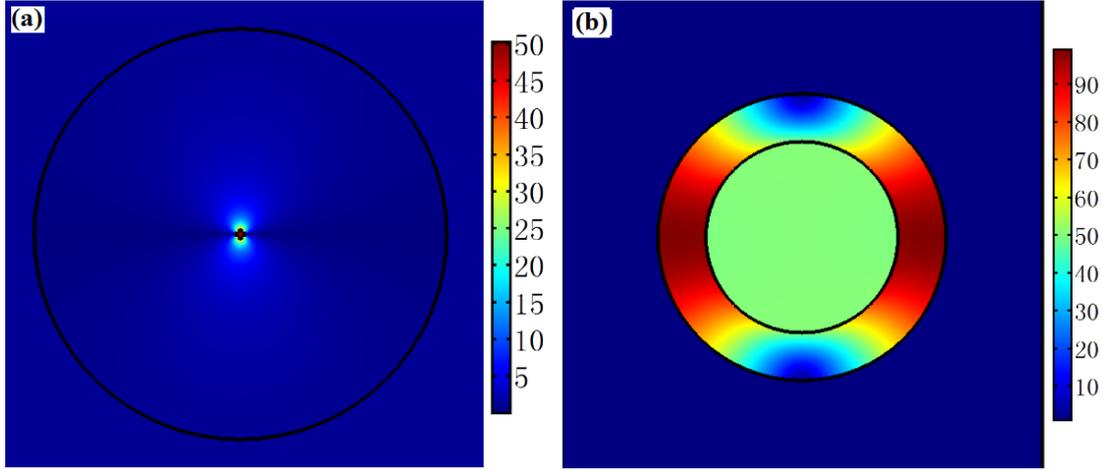


Fig. 2: (FEM simulation result) The magnetic flux density distribution in 2D space when we apply a uniform background magnetic flux density $B=1\text{T}$ in y direction for different devices: (a) the magnetic energy harvesting device proposed by [26]. As we can see here if we want to realize a 50T field in a small air region $R_1=0.1\text{m}$, we have to use an extremely large device whose outer size is $R_2=1\text{m}$ and inner size is $R_1=0.1\text{m}$. (b) the super-concentrator with $R_1=0.2\text{m}$, $R_2=0.3\text{m}$ and $R_3=20\text{m}$. We have a uniform magnetic field 50T inside the device (it is free space). We should note that R_3 is a virtual size which affects the degree of enhancement and material distribution of device. The geometrical size of device in (b) is totally determined by R_1 and R_2 .

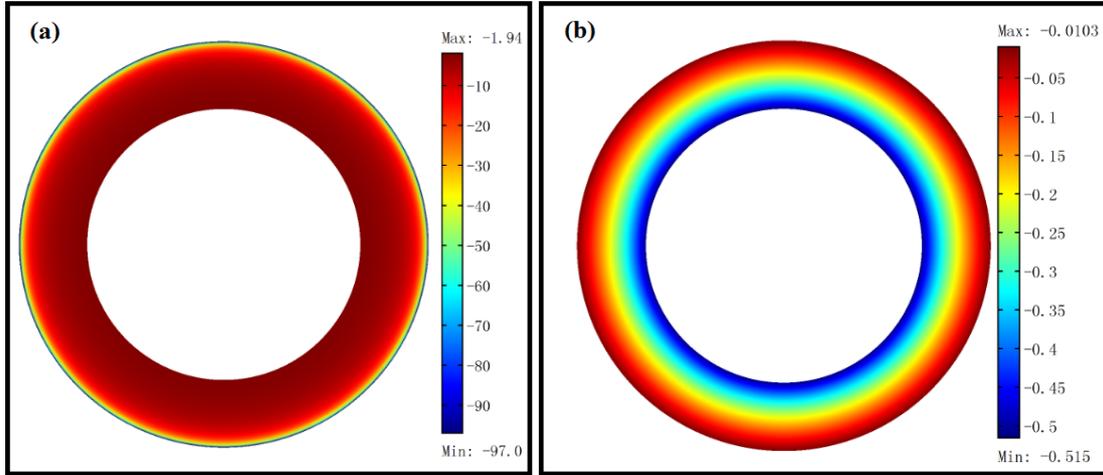


Fig. 3: The permeability of a super-concentrator for static magnetic field in tangential direction μ_θ (a) and in radial direction μ_ρ (b). The materials in (a) and (b) is for the device in Fig. 2(b) with $R_1=0.2\text{m}$, $R_2=0.3\text{m}$ and $R_3=10\text{m}$.

4. Summary

A compact passive device that can be used to realize a larger than 50T uniform static magnetic field in a large 2D free space has been designed by transformation optics and verified by the finite element method. This device will have potential applications in many areas which require high magnetic fields (e.g., magnetic resonance imaging, etc.) over a large area. Compared with recent methods for enhancing static magnetic fields, our device is passive and compact, and is made of negative anisotropic inhomogeneous permeability, which can be realized by magnetic metamaterials.

5. Reference

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