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In situ determination of the static inductance and resistance of a plasma focus capacitor bank

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The static (unloaded) electrical parameters of a capacitor bank are of utmost importance for the purpose of modeling the system as a whole when the capacitor bank is discharged into its dynamic electromagnetic load. Using a physical short circuit across the electromagnetic load is usually technically difficult and is unnecessary. The discharge can be operated at the highest pressure permissible in order to minimize current sheet motion, thus simulating zero dynamic load, to enable bank parameters, static inductance L_0, and resistance r_0 to be obtained using lightly damped sinusoid equations given the bank capacitance C_0. However, for a plasma focus, even at the highest permissible pressure it is found that there is significant residual motion, so that the assumption of a zero dynamic load introduces unacceptable errors into the determination of the circuit parameters. To overcome this problem, the Lee model code is used to fit the computed current trace to the measured current waveform. Hence the dynamics is incorporated into the solution and the capacitor bank parameters are computed using the Lee model code, and more accurate static bank parameters are obtained. © *2010 American Institute of Physics.* [doi:10.1063/1.3429207]

I. INTRODUCTION

To model any electromagnetic device, among the input parameters are the static bank parameters. For example for any modeling of a plasma focus discharge (see Fig. 1), among the input parameters are the static inductance L_0 and resistance r_0, given the bank capacitance C_0. The word static is used here in preference to the phrase "short circuit" as the former conveys the more accurate meaning that these parameters are the relevant parameters when there is no dynamic electromagnetic load. Moreover static also implies *in situ*; that everything remains the same except that there is no motion attributed to the current distribution. The term short circuit is not sufficiently accurate in this context, as the position and design of the short circuit are subject to sometimes arbitrary definition which would give unreliable results.

A simple approximate method is to operate the plasma focus at high pressures, say 20 Torr neon; so that there is little current sheet motion. Then the discharge current approximates a lightly damped L_0-C_0-r_0 oscillation where C_0 is the bank capacitance and L_0 and r_0 are the static inductance and resistance of the bank, respectively. Analysis of two or three cycles of the current waveform then yield approximate values of L_0 and r_0, given that C_0 is known.

However it turns out that even at 20 Torr neon, there is usually significant motion of the current sheet which affects the current waveform, as this motion introduces an additional time-varying inductance into the circuit; even though at such high pressures the axial motion is not fast enough to reach the end of the anode at the end of the first half cycle drive. So there is no focus (radial) phase (see Fig. 1). Nevertheless, because of the axial motion, the estimated static inductance will be too high. Therefore an analysis of the current waveform including the small but significant current sheet dynamics is necessary in order to obtain a true measure of L_0. In this paper we discuss the estimate of capacitor parameters using a high pressure discharge first using damped L_0-C_0-r_0 analysis, assuming no current sheet dynamics. Then we show that actually there is significant current sheet motion. Finally we use a plasma focus code, with included dynamics to analyze the measured current waveform, thus separating the L_0 and the additional amount of inductance due to motion.

II. EXPERIMENTAL PROCEDURES AND RESULTS

A. Estimate of capacitor bank static parameters from a high pressure discharge

An example is given here, using the INTI-PF, which is one of the plasma focus machines developed from the UNU/ICTP PFF.[1,2] The capacitor bank has a capacitance of $C_0 = 30\ \mu F$. The plasma focus is filled with neon at 20 Torr. The capacitor bank is charged up to 10 kV and discharged. The current trace is monitored with a current transformer. The discharge current waveform is as shown in Fig. 2.

Assuming that the discharge current waveform is that from a lightly damped L_0-C_0-r_0 circuit, the waveform may be treated as sinusoid with period T; the following approximate equations[3] hold:

$$L_0 = T^2/(4\pi^2 C_0), \tag{1}$$

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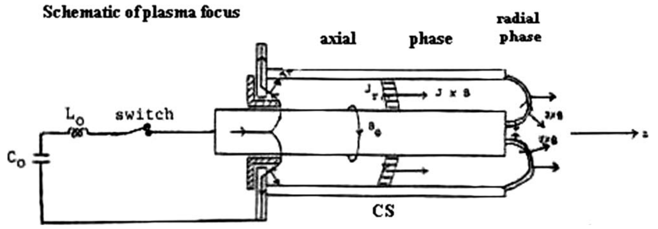

FIG. 1. Showing the plasma focus with representative current sheet positions in the axial phase and the radial phase. The plasma focus acts in the following way: A capacitor bank C_0 discharges a large current into the coaxial tube. The current flows in a current sheath CS which is driven by the $J \times B$ force, axially down the tube. At the end of the axial phase the CS implodes radially, forming an elongating pinch. The static resistance r_0 of the discharge circuit is not shown.

$$r_0 = -(2/\pi)[\ln(f)](L_0/C_0)^{0.5}, \tag{2}$$

$$I_0 = \pi C_0 V_0(1+f)/T, \tag{3}$$

where f is the reversal ratio obtained from the successive current peaks I_1, I_2, I_3, I_4, and I_5 with $f_1 = I_2/I_1$, $f_2 = I_3/I_2$, $f_3 = I_4/I_3$, $f_4 = I_5/I_4$, and $f = (1/4)(f_1+f_2+f_3+f_4)$; and I_0 the highest peak current is written here as I_1, the peak current of the first half cycle.

From Fig. 2, we estimate the following:

$$3T = 36.6\ \mu\text{s},$$

giving $T = 12.2\ \mu$s and

$$L_0 = 126\ \text{nH}.$$

$f_1 = 0.737$, $f_2 = 0.612$, $f_3 = 0.760$, and $f_4 = 0.524$, giving $f = 0.658$; and

$$r_0 = 17.1\ \text{m}\Omega$$

and peak current $I_0 = 128$ kA.

The coil gives a peak first half cycle output of 24 V. Thus additionally the coil sensitivity is obtained as 128/24 = 5.3 kA/V.

B. Correction required if the current sheet had moved

If the current sheet had moved, then the movement will add to the circuit inductance; and the value of the inductance measured will have a part which is due to inductance increment as a result of motion. To estimate whether there is significant motion at this pressure we fire a shot at a low enough pressure to obtain a strong focus in order to obtain the axial speed at the lower pressure. This is shown in Fig. 3.

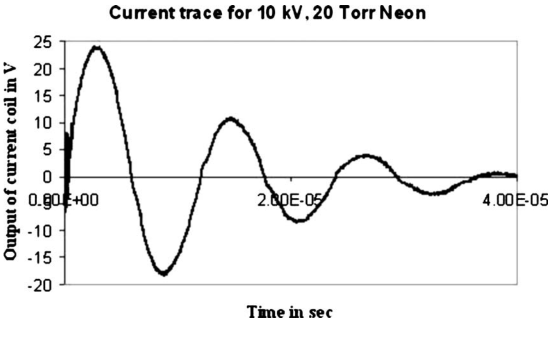

FIG. 2. Measured discharge current waveform at 10 kV, 20 Torr neon.

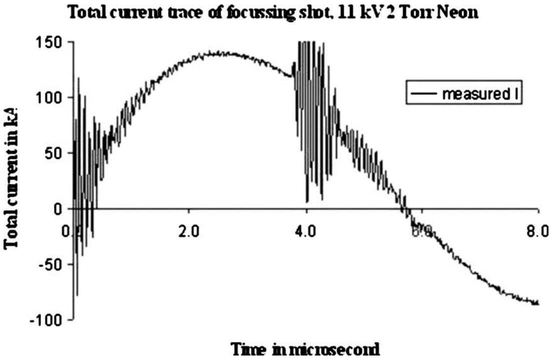

FIG. 3. Discharge current waveform at 11 kV, 2 Torr neon.

From this shot we deduce the following. The current dip starts at the point which is approximately the end of the axial phase and the start of the radial phase.

So it takes 3.9 μs for the current sheet to travel the 16 cm which is the distance traveled by the current sheet over the full length of the axial phase. The average speed of the current sheet down the tube in this axial phase is 4.1 cm/μs for this shot at 11 kV, 2 Torr neon.

From these data we can estimate the average speed of the high pressure shot at 10 kV 20 Torr neon; using the scaling relation[4] for electromagnetic drive

$$\text{speed} \sim (I/a)/\rho^{0.5}.$$

The anode radius a is constant, and the current is approximately proportional to the charging voltage. Here ρ is the density which is proportional to the pressure for a fixed gas. Hence the average axial speed for the 10 kV, 20 Torr neon shot is estimated from the relationship to be $(10/11)(2/20)^{0.5}(4.1) \sim 1.2$ cm/μs.

So in the time of the first half cycle of 6.1 μs, the current sheet would have moved 7 cm. The coaxial tube has an inductance of ~2.4 nH/cm. So the current sheet movement could have added a maximum amount of 17 nH to the measured inductance of the high pressure shot. This is not a negligible amount. Hence the motion should be taken into account for the measurement of L_0.

C. Fitting the computed current trace to the measured current trace to determine the static inductance

The Lee model code[5] couples the electrical circuit with plasma focus dynamics, thermodynamics, and radiation, enabling realistic simulation of all gross focus properties.[6–10] For the high pressure shot involving only the axial phase, the code is very precise in its computation of the discharge current, including the interaction of the circuit with the current sheet dynamics. The Lee model code is used in order to generate a computed current trace for fitting to the measured current trace. In this fitting, adjustments are normally made to four model parameters, f_m, f_c for the axial phase, and f_{mr} and f_{cr} for the radial phase. In this case we are fitting the 20 Torr neon shot, which has no radial phase, since the current sheet does not move fast enough to reach the end of the anode during the drive time of the current pulse.

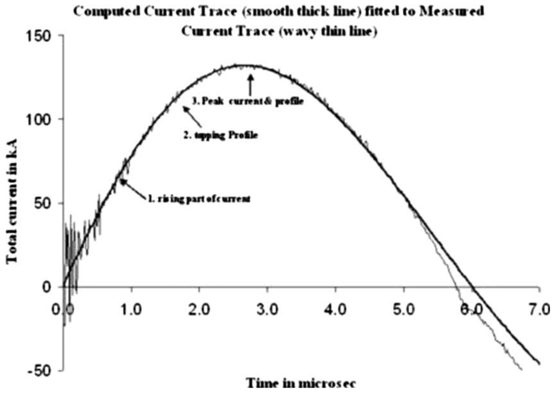

FIG. 4. Fitting the computed current trace to the measured current trace by varying model parameters f_m and f_c.

Our experience has shown us that the three features to be fitted for the axial phase, i.e., the rising part of the current trace, the topping profile, and the peak current have to be fitted by adjusting the value of f_m and f_c (see Fig. 4).

The method is also sensitive enough that the static inductance and circuit stray resistance also need to be correctly fitted; otherwise no good fit would be obtained. Further, for some low voltage shots, it may also be necessary to move the whole measured current trace a small amount in time; otherwise no good fit may be obtained. This is due to the nonperfect switching characteristics of the spark gap which typically takes a little time to go from nonconducting to conducting, whereas the code assumes instantaneous switching at time zero. This is apparent when the early rising slope of the high pressure shot is examined. The rising section of the measured current does not rise as fast as the computed current trace no matter how the model parameters f_m and f_c are adjusted and no matter how L_0 and r_0 are adjusted. The correct way is to shift either current trace relative to the other by the addition of a small time delay.

Figure 4 shows the results of the fitting of the computed current trace to the measured trace according to the procedures summarized in the earlier paragraph, and discussed in greater detail below. We start the fitting process by using the static inductance and resistance estimated earlier assuming no plasma dynamics; i.e., $L_0=126$ nH and $r_0=17.1$ mΩ, with capacitance $C_0=30$ μF. Since for this high pressure shot the current sheet does not move beyond the end of the anode, we only need the first two of the model parameters, which we initialize by taking $f_m=0.08$ and $f_c=0.7$.

By comparing the computed with the measured current waveforms, it was clear that the first part of the current trace was not going to fit except by shifting the whole measured current trace. This was done, and the required shift was 0.2 μs to make the measured current trace come earlier in time relative to the computed. Next it was found that no suitable fit could be found unless we change the value of L_0 to $L_0=114$ nH, $r_0=15$ mΩ. Finally in small incremental steps the following values of model parameters (axial phase) were found to be $f_m=0.05$ and $f_c=0.71$. The resultant fit is shown in Fig. 4.

III. DISCUSSION

The results of this method of analysis for this 10 kV, 20 Torr neon shot are the following: bank parameters: $L_0=114$ nH, $C_0=30$ μF, and $r_0=15$ mΩ; model parameters (axial phase only): $f_m=0.05$ and $f_c=0.71$; tube parameters: $b=3.2$ cm, $a=0.95$ cm and $z_0=16$ cm. Here b and a are the outer and inner radii of the coaxial tube and z_0 is the length of the anode. At the same time the Rogowski coil (used as a current transformer) was calibrated with a sensitivity of 5.4 kA/V.

In obtaining the best fit as shown in Fig. 4, it was noted that for $t>5.0$ μs, there is a feature in the measured current trace which is not present in the computed current trace. The computed current trace assumes that the current sheet continues to move axially down the tube. However it is known that in electromagnetic shock tubes, which the axial part of the plasma focus is, the voltage across the tube reverses just before the current changes direction in the external circuit. At that point of time when the voltage changes direction, the anode to cathode current starts to close upon itself to form a closed loop.[11] With this loop decoupled electrically from the capacitor, the inductance of the capacitor discharge circuit is reduced. This causes the external current to drop quicker to zero and the modeled current trace, which does not have this closing loop feature, is no longer able to be fitted to the measured current trace. We include this discussion here to show how sensitive is the method of fitting modeled current to measured current.

IV. CONCLUSION

In this paper, a two-step method is discussed to determine the capacitor bank static parameters of a plasma focus. In the first step of the estimate, the assumption is made that there is no current sheet movement for the high pressure discharge. Hence the discharge current may be analyzed by equations that assume a lightly damped sinusoid generated from an L_0-C_0-r_0 discharge circuit, where C_0 is known, and L_0 and r_0 are constant in time and are thus easily determined from Eqs. (1) and (2). The second step takes into account the current sheet motion. This step involves fitting the current trace computed with the Lee model code to the measured current trace, using the estimated values of L_0 and r_0 obtained from the first step. For this fitting, it is found that the static inductance L_0 has to be adjusted before the fitting is possible, with the adjustment of r_0 also playing a significant role. Finally the model parameters have to be adjusted for the best fit.

This two-step process enables the values of L_0 and r_0 to be correctly measured. At the same time the current measuring device is also more accurately calibrated.

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