IMAGING SPECTROPOLARIMETRY OF Ti I 2231 nm IN A SUNSPOT

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(Received 20 December 2002; accepted 26 February 2003)

Abstract. Spectro-polarimetric observations at 2231 nm were made of NOAA 10008 near the west solar limb on 29 June 2002 using the National Solar Observatory McMath–Pierce Telescope at Kitt Peak and the California State University Northridge – National Solar Observatory infrared camera. Scans of spectra in both Stokes I and Stokes V were collected; the intensity spectra were processed to remove strong telluric absorption lines, and the Stokes V umbral spectra were corrected for instrumental polarization. The sunspot temperature is computed using the continuum contrast and umbral temperatures down to about 3700 K are observed. A strong Ti I line at 2231.0 nm is used to probe the magnetic and velocity fields in the spot umbra and penumbra. Measurements of the Ti I equivalent width versus plasma temperature in the sunspot agree with model predictions. Zeeman splitting measurements of the Stokes I and Stokes V profiles show magnetic fields up to 3300 G in the umbra, and a dependence of the magnetic field on the plasma temperature similar to that which was seen using Fe I 1565 nm observations of the same spot two days earlier. The umbral Doppler velocity measurements are averaged in 16 azimuthal bins, and no radial flows are revealed to a limit of $\pm$ 200 m s^{-1}. A Stokes V magnetogram shows a reversal of the line-of-sight magnetic component between the limb and disk center sides of the penumbra. Because the Ti I line is weak in the penumbra, individual spectra are averaged in azimuthal bins over the entire penumbral radial extent. The averaged Stokes V spectra show a magnetic reversal as a function of sunspot azimuthal angle. The mean penumbral magnetic field as measured with the Stokes V Zeeman component splitting is 1400 G. Several weak spectral lines are observed in the sunspot and the variation of the equivalent width versus temperature for four lines is examined. If these lines are from molecules, it is possible that lines at 2230.67, 2230.77, and 2231.70 nm originate from OH, while the line at 2232.21 nm may originate from CN.

1. Introduction

The Ti I 2231 nm absorption line is one of several titanium lines near 2200 nm which are visible in sunspot spectra. Systematic polarization measurements of these lines were first done by Rüedi *et al.* (1998). In that work, using the National

*Operated by the Association of Universities for Research in Astronomy, Inc. (AURA), under cooperative agreement with the National Science Foundation.

Solar Physics **215:** 87–97, 2003.

Solar Observatory (NSO) McMath–Pierce Telescope at Kitt Peak and data from the FTS instrument and slit spectra from the NIM instrument, Rüedi *et al.* found two unique magnetic components in a sunspot, and an abrupt change between these two fields at the umbral–penumbral boundary as well as a signature of an Evershed outflow. That work also computed line strengths and polarization profiles using a model atmosphere. Subsequent work by Rüedi, Solanki, and Keller (1999) showed that in the cool penumbral filaments the Ti I 2231 nm spectropolarimetry revealed a weak and rather constant magnetic field strength between 500 and 900 G with an inclination to solar vertical of almost 90 deg. In the present work full-imaging spectropolarimetry is used to investigate the umbral and penumbral velocity and magnetic fields as well as the behavior of line strength with plasma temperature.

Instrumental polarization presents difficulties in interpreting spectro-polarimetric data. For lines in which the Zeeman splitting is fully resolved removal of instrumental contamination is facilitated. Work by Kuhn *et al.* (1994) describe an approach to remove instrumental polarization from spectral measurements of all four Stokes parameters of the Fe I 1564.8 nm line. Although in the data described in this paper only measure Stokes I and Stokes V spectra of Ti I 2231 nm, a modified technique based on the approach used by Kuhn *et al.* is employed to remove polarization crosstalk to below 0.01 times the continuum intensity.

The temperature dependence of the total magnetic field strength as measured by the Zeeman splitting of the Stokes I components of 1564.8 nm and the 630.2 nm Fe I lines from work published by several authors was compared recently by Penn *et al.* (2003). By accounting for the difference in the calculated temperature caused by different heights of formation of the continuum near each line, and the magnetic field difference computed using the line formation heights and a typical vertical magnetic gradient the results from the visible and near infrared lines were found to agree within the measurement errors. For the first time an analysis of the magnetic field strength versus temperature (temperature calculation is discussed in Section 3.4) is done in this work using the Ti I 2231 nm line ($g_{\text{eff}} = 2.5$) and this is compared with measurements of the same sunspot using the 1564.8 nm Fe I line ($g_{\text{eff}} = 3$) made a few days prior to the Ti I observations.

2. Observations and Reduction

The NSO McMath–Pierce telescope is described in detail by Pierce (1964) and infrared observations have been made with the main spectrograph by a number of authors. In 1992 a large infrared grating (47 $\times$ 37 cm) was installed in the spectrograph; these observations used the main heliostat feeding the main spectrograph with this infrared grating. Telescope scanning was done by changing the main heliostat pointing. The joint California State University Northridge (CSUN) – NSO infrared camera is a HgCdTe based system, using a 256 $\times$ 256 pixel detector and was described in Penn *et al.* (2003). In these observations the spectral dispersion was 0.0073 nm per pixel and 0.4 arc sec per pixel along the spectrograph slit, and the telescope was scanned in 0.4 arc sec steps perpendicular to the slit. The slit was oriented roughly parallel to the local solar limb. The seeing and image motion effectively limits the spatial resolution to about 1 arc sec. The spectral field of view covered about 2 nm and the spatial scans covered about 85 by 56 arc sec and no continuum changes due to center-to-limb variations were seen in the small 56 arc sec radial direction. No spatial or spectral binning or smoothing was done. For the polarization analysis a pair of Meadowlark Optics liquid crystal variable retarders were used in conjunction with a linear polarizer to modulate the retardance between $\pm\frac{1}{4}$ wave retardance during exposures. Each retarder was found to only be capable of providing $\frac{1}{4}$ wave of retardance, and so the fast axis of the two retarders were oriented orthogonally and each was set to either zero or $\frac{1}{4}$ wave of retardance during the $I + V$ and $I - V$ exposures. The retarders were modulated at about 4 Hz and the effective integration at each slit position is roughly 1 s. The signal-to-noise ratio in the raw spectra is on the order of 100.

A pair of $I+V$ and $I-V$ frames was averaged to produce an intensity spectrum. The intensity spectra were dark subtracted in the usual way. A spectral flat field was produced from 40 quiet-Sun slit positions within each sunspot scan and processed using an algorithm similar to Jones (1999). A continuum fit using a quadratic function outside of the telluric and Ti I lines was used to remove continuum variations and to make the continuum intensity maps. After dividing by this continuum function, an averaged telluric absorption spectrum was produced from the quiet-Sun slit positions within each sunspot scan. At each slit position in each frame, this mean spectrum was fit to the observed spectrum using the continuum between 2231.4 and 2231.7 nm and the depth of the strongest telluric line at 2231.0 nm. The observed intensity spectrum was then divided by this scaled telluric spectrum for the final corrected intensity spectrum. It is interesting to note that the line strength of the telluric line normalized by the continuum appears to change across the scan field of view, increasing in strength (normalized by the continuum) in the darkest parts of the umbra; this cannot be explained by scattered photospheric light or spectrally scattered light in the spectrograph. This was found to be true for both the 2231.0 and the 2231.75 nm telluric lines and suggests that there may be hidden umbral lines at wavelengths close to these telluric lines. Figure 1 shows a sample intensity spectrum through various stages of this correction process. It is important to note that no photospheric absorption lines are seen in this region of the spectrum.

The Stokes V spectra were reduced and calibrated in a different manner. The Stokes V spectral frames are formed by simply subtracting a pair of images, one taken while the polarimeter optics passed $I + V$ to the spectrograph, and the other image while the polarimeter transmitted $I - V$. This difference is divided by two. No dark subtraction is done and no flat fielding is used. This raw Stokes V frame (see Figure 2) contains vertical streaks of continuum polarization which are especially strong in regions where the intensity gradient along the spectrograph slit is large; these streaks are attributed to either image motion or different seeing

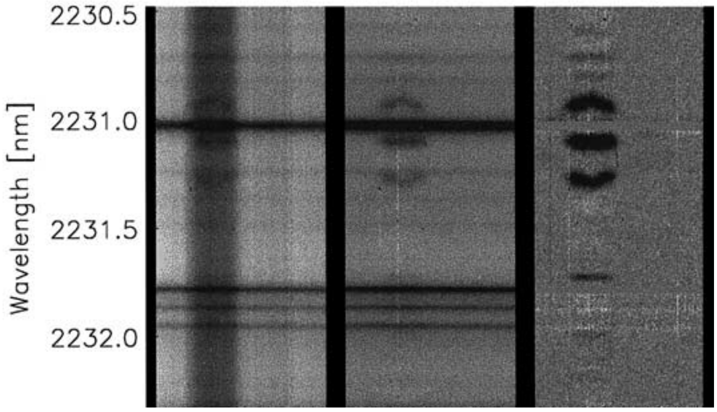

Figure 1. Processing of Stokes I spectra. The *left panel* shows a flat-fielded spectrum where the slit crosses the umbra of NOAA 10008. In the *middle panel* the continuum variations are removed by division and the Ti I line multiplet becomes obvious. In the *right panel* the telluric absorption lines are removed by division, and the regions in the cores of the strongest lines (2231.0 and 2231.78nm) are suppressed. Note that there are no photospheric lines in this part of the spectrum; the Ti I line and a few weaker lines appear in the umbral and penumbral regions.

between the two individual exposures. A fit is made to the polarization signal at continuum wavelengths and the fit is simply subtracted.

The central π component seen in these continuum corrected Stokes V frames (V_π, which should be zero) was used to remove the instrumental polarization. A first attempt was made assuming all the crosstalk is Stokes I to Stokes V; however this correction produced highly asymmetric Stokes V profiles, many containing only a red or a blue component. A map of the central π component observed in Stokes V was made and found to be unlike the Stokes I map, but very similar to a Stokes U map of the sunspot made using the Fe I 1565 nm line on 27 June 2002. Thus it seems the instrumental contamination of the Stokes V frames is dominated by Stokes U rather than by Stokes I. This introduces a problem, since no linear polarization measurements were made at Ti I 2231 nm due to limitations of the polarimeter set-up. So while the technique followed by Kuhn *et al.* (1994) can determine the inverse Mueller matrix using the fully split Zeeman profiles, without full Stokes measurements the Mueller matrix cannot be determined, and the Stokes V profiles must be corrected at each spatial position using a proxy for the linear polarization spectrum.

It is possible to develop an artificial Stokes Q or Stokes U profile using the a Stokes I profile only if the inclination and azimuth angles of the magnetic field are known. Basically, the Stokes U profile will have a central π component with oppositely signed split σ components compared to the Stokes I profile, or more precisely compared to $I_C - I_\lambda$ where I_C is the continuum intensity. However the amplitude of the central Stokes U π component (U_π), and the ratio of the amplitudes of the Stokes U π and σ components (U_π/U_σ) depend on the magnetic field inclination and azimuth angles (for example, Sakurai (2000) develops the expression for Q_π/Q_σ). In these observations, the amplitude of the contaminating U_π component is determined by scaling the Stokes I spectrum to the observed V_π at each spatial pixel. However, the ratio of U_π/U_σ remains unknown, and so the artificial Stokes U spectrum is created with U_π/U_σ set equal to I_π/I_σ. While this is exactly correct for only one given field orientation (where the field is oriented perpendicular to the line of sight) the approximation is fairly good for this sunspot, where a locally vertical field in the sunspot is seen close to the solar limb. The problem of using the wrong U_π/U_σ ratio will introduce an artificially asymmetry into the Stokes V profiles as the V_σ components are either under or over corrected. As shown in the last panel of Figure 2, the technique does an excellent job of removing the spurious V_π component, and also the strange symmetric components seen in the uncorrected Stokes V profile. A closer examination of the Stokes V profiles shows that most profiles have a very small residual asymmetry.

The final correction step made to the Stokes V profiles is to compute the median pixel value for all corrected Stokes V frames and to subtract this value. In this median frame, a weak Stokes I spectrum is observed with an amplitude of roughly 0.01 times the continuum intensity. As most of the V_π profile amplitudes are at the level of 0.3 of the continuum intensity, this further substantiates that Stokes U contamination dominates Stokes I contamination by a factor of about 30. A second component in this median frame is seen which resembles a pattern of bright fringes with an amplitude of roughly 4×10^{-3} of the continuum intensity perhaps caused by the polarimeter optics. Both the Stokes I contamination and the fringes are removed from the Stokes V by subtracting this median frame.

3. Discussion

Figure 3 shows several maps of the sunspot region. On 29 June NOAA 10008 was located near the west limb at $\mu = 0.39$. First the continuum intensity is plotted, and then the total strength of the Ti I line. Absorption from the Ti I line is seen in the umbra and also in the penumbra of the sunspot. Also shown are maps of the total magnetic field strength as determined from the wavelength splitting, and the Stokes V line-of-sight component of the magnetic field.

3.1. Umbral Magnetic Fields

The Stokes I σ components were fit with simple Gaussian functions, and the wavelength difference between these two components was used to compute the total magnetic field in the sunspot. Weak line strength prevented useful fits above a temperature of 5400 K. However, below that temperature we plot the total magnetic field versus the plasma temperature in Figure 4.

The relationship for the Ti I line is identical to the Fe I 1565 nm magnetic field and temperature relationship as observed on 27 June 2002 except the Ti I curve is

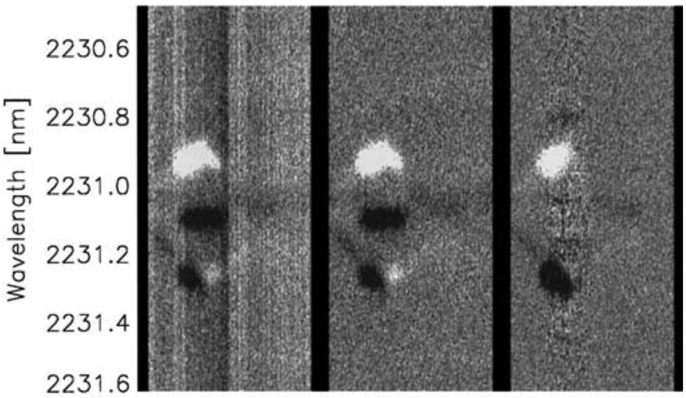

Figure 2. Processing of Stokes V spectra using the Ti I line at 2231 nm. The *left-most frame* is the difference of the $I - V$ and $I + V$ frames and contains vertical streaks caused by image motion between the time of exposure of the two frames. The vertical streaks are subtracted in the *middle frame*; however the *middle frame* still shows an unshifted π line component in the sunspot umbra, from cross-talk with other Stokes states due to instrumental polarization. This component is removed with a technique similar to Kuhn *et al.* (1994), and lower-order systematic effects are also removed by subtracting a median Stokes V frame from the quiet Sun (see text) to produce the corrected *frame on the right*.

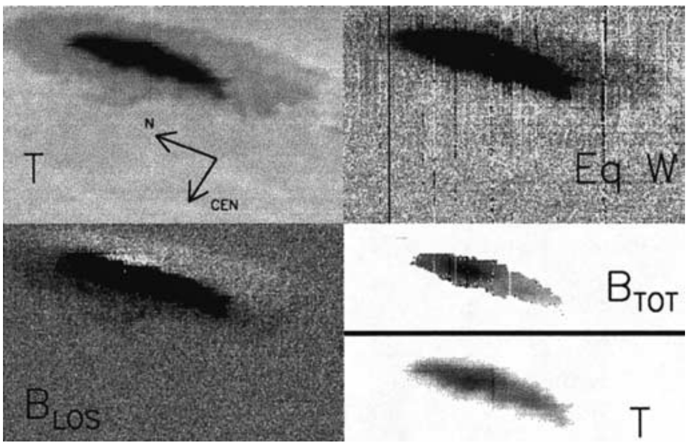

Figure 3. Five images of the NOAA 10008 spot region, *clockwise from upper left:* temperature, Ti I equivalent width, umbral total magnetic field, and temperature maps and line-of-sight magnetic component. Each image is oriented with solar north to the upper left corner, the solar limb direction up and the solar disk center direction down. The field of view of the three large images is about 85 arc sec by 56 arc sec. The temperature map is scaled from 4500 K (*black*) to 6200 K (*white*), the equivalent width is reverse contrast scaled from 50 mÅ to −15 mÅ to show Ti I in the penumbral regions. The *lower right panel* shows the total magnetic field strength measured from splitting of Stokes I scaled from −3300 to −2000 G, and also a temperature map scaled from 3500 to 5500 K. The *lower left panel* is the integral of Stokes V and arbitrarily scaled to show the reversal of the penumbral fields.

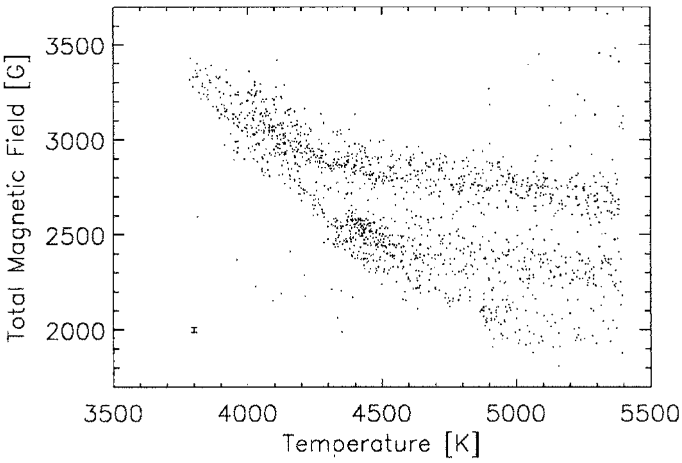

Figure 4. Total magnetic field versus temperature for NOAA 10008. The temperature is determined from the continuum contrast by assuming a simple Planck function, and the total magnetic field is measured with the Zeeman splitting of the Ti I σ components. All measured points lie within the sunspot umbra, as the Ti I becomes too weak to directly measure the splitting at higher temperatures. Possible causes for the range of magnetic field strengths seen at temperatures from 4200 K to 5400 K are discussed in the text. The expected measurement error for the magnetic field is small and is shown by the mark in the lower left corner of plot.

shifted to lower temperatures by 500 K, and to higher magnetic fields by 100 G. The difference in the continuum formation heights at these two wavelengths suggests that the 2231 nm temperatures should be cooler than the 1565 nm temperatures by about 100 K (Vernazza, Avrett, and Loeser, 1976). Also the formation height of the 2231 nm Ti I line is higher than the 1565 nm Fe I line by 100 km (Rüedi *et al.*, 1998) and assuming a vertical magnetic field gradient of 2 G km^{-1} (Pahlke and Wiehr, 1990) implies that the magnetic field measured by Ti I should be 200 G weaker than that measured by Fe I, in direct contrast to the observations. It is likely that there were changes in the sunspot magnetic field between 27 June and 29 June which would preclude this direct comparison; simultaneous observations at these two wavelengths would remove this uncertainty.

The bimodal structure of the magnetic field and temperature relationship at temperatures between 4300 K and 5500 K is also seen in the Fe I 1565 nm observations. In Figure 5 the spatial locations of the pixels are shown. The northern part of the umbra of NOAA 10008 is the location of the pixels which have the higher magnetic field strengths for a given temperature, while the southern parts of the NOAA 10008 umbra have a lower magnetic field strength for a given plasma temperature. No correction for large angle scattered or seeing induced stray light was done with this data, and because the northern pixels lie in a region with a steep temperature gradient (see Figure 3, lower right image) it is possible that seeing effects could increase the measured continuum temperature at those locations by adding penumbral or photospheric light. Also, Kopp and Rabin (1992) point out

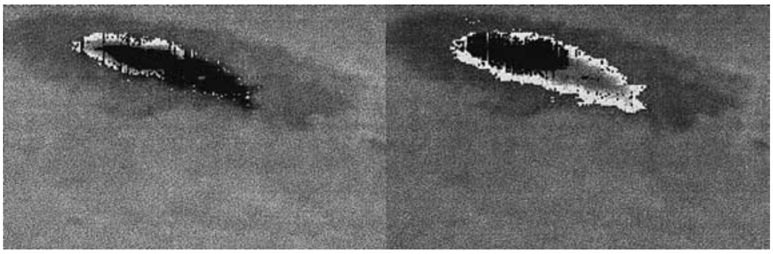

Figure 5. Two maps of NOAA 10008 showing regions with different magnetic field versus temperature relationships. The orientation is the same as in Figure 3. The *left image* shows the location of pixels (*bright*) with magnetic fields greater than 2600 G and temperatures between 4300 K and 5500 K, while the *right-hand image* shows the location of pixels (*bright*) in the same temperature range but with a total field strength less than 2600 G. See text for discussion.

that a distinct relationship between magnetic field strength and plasma temperature is only possible assuming a simple axi-symmetric magnetic field geometry, and for example a field with a transverse component may not show the same relationship. Significantly complex geometry of umbral magnetic fields have been seen in other observations (i.e., Leka *et al.*, 1996) and it is possible that similar complexity in NOAA 10008 may explain the observed bimodal structure.

3.2. Penumbral magnetic fields

The vector components of the magnetic field can be explored by comparing the integrated Stokes V signal (which measures the line-of-sight (LOS) magnetic component) with the Zeeman splitting of the Stokes I signal (which measures the total magnetic field). A map of the ratio of the LOS component to the total field shows an enhancement on the disk-center side of the sunspot umbra. This supports a simple model of the spot magnetic field where the inclination of the sunspot magnetic field increases with radial distance from the umbral center.

Finally the Zeeman splitting in the Stokes I data was compared with similar measurements (simple Gaussian fits of the σ components) of the Stokes V data. As expected, the Stokes V measurements have lower signal-to-noise ratio, but where the fits are good the two Zeeman splittings show a one to one correspondence.

Figure 3 shows a reversal of the line-of-sight component in the disk center sunspot penumbra compared to the limb-side penumbra in the magnetogram type of image. Because the spectral profiles of the Ti I line are relatively weak it is not possible to fit the Stokes V profiles at each pixel in the penumbra. Since the magnetogram reversal is well fit by the sine of the sunspot azimuthal angle, a binning process was done with the raw spectra to improve the signal to noise ratio. The penumbra was defined with continuum temperatures between 5200 K and 5850 K, and the reduced spectra from every pixel in azimuth bins of $\pi/16$ radians were averaged to produce a mean penumbral spectrum as a function of azimuth angle. These averaged spectra show the magnetic reversal in the penumbra between the limb and disk center sides, and also show that the Stokes V component weakens toward solar north and south, in directions parallel with the solar limb. These findings are consistent with the interpretation that the penumbral magnetic field is highly inclined in the cool penumbral plasma as measured by the Ti I line. The mean field through the penumbra is 1400 G, and any radial variation within the penumbra is not apparent.

3.3. Sunspot flows

The umbral plasma velocity can be computed with the mean wavelength of the two split σ components. As the spot is near the limb any horizontal flows within the umbra would manifest themselves as an azimuthally varying Doppler shift. Radially binning across the whole umbra and plotting the velocity of the magnetic component versus azimuth shows no sinusoidal behavior. The limit on the amplitude from the noise in this plot is ± 200 m s^{-1} for horizontal outflows or inflows in the umbra.

3.4. Ti I equivalent width and temperature

The temperature at each point in the scan is determined by assuming the quiet Sun continuum has a temperature of 6000 K at 2231 nm, and using the continuum normalized intensity and the Planck function as in Penn *et al.* (2003). The sunspot umbra contains regions with plasma temperatures between roughly 4000 and 5500 K, the penumbra from 5500 to 5800 K, and the surrounding photosphere shows plasma temperatures in the 5800 to 6100 K range. The equivalent width of the Ti I line was computed by integrating over all three components in the telluric corrected intensity spectra. The result is shown in Figure 6. Also shown in Figure 6 is a measurement taken by one of us using a different grating and a bolometer detector (see Livingston, 2002) of a sunspot NOAA 9973 on 4 June 2002. In those data only the equivalent width of one of the σ components is measured. Assuming that each Zeeman σ component and the unshifted π component have the same equivalent width, the total equivalent width can be determined by multiplying the width of one σ component by three. With this very simple approximation, and with a different instrumental setup, a different detector and a different sunspot the measurements agree within the scatter.

Rüedi *et al.* (1998) predict the behavior of the Ti I 2231 nm equivalent width versus temperature by using a model atmosphere (see their Figure 3). A line showing their results is also plotted in Figure 6, where we interpret their calculations to represent the equivalent width of one of the split Zeeman components, so we multiply their widths by a factor of three. The equivalent widths at the lowest temperatures seen in this sunspot are not inconsistent with the predicted drop in Ti I equivalent width at temperatures below 3900 K (Rüedi *et al.*, 1998) due to formation of the TiO molecule. The correlation is very good in the penumbra and in the coldest umbral regions, but the measurements seem to be systematically higher

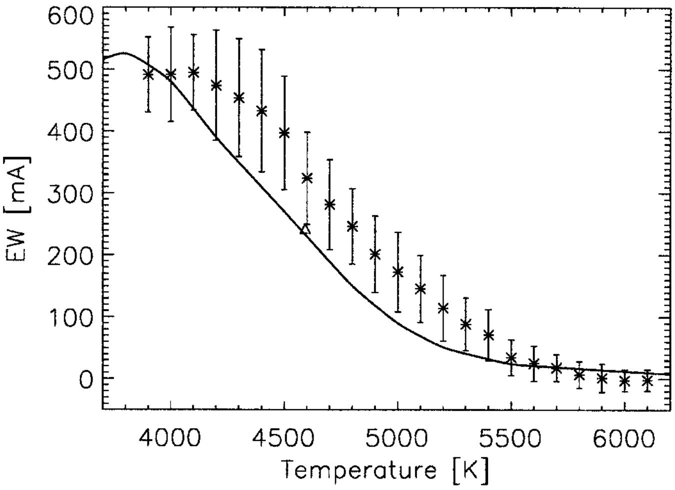

Figure 6. Equivalent width of the Ti I 2231 nm line compared with the model calculation from Rüedi *et al.* (1998). The temperature is measured from the continuum contrast; the measurements were binned in 100 K bins and the error bars show the standard deviation within each bin. Also shown with the triangle at 4600 K is a measurement from the Baboquiviri detector from a different sunspot (see text).

than the model predictions at temperatures between 4200 and 5500 K. Contamination from incorrect removal of the strong telluric line at 2231.0 nm is ruled out as summing over only the red component and the red-half of the central component and multiplying by two gives identical results. The cause of this slight discrepancy remains unknown.

3.5. Weak Spectral Lines

There are four faint spectral lines at wavelengths close to the Ti I 2231 nm line which can be measured with the current data, and there are suggestions of even weaker lines at other wavelengths. None of these lines are identified in the spectral atlas of Wallace, Hinkle, and Livingston (2001, hereafter WHL). The H_2O line listed by WHL at about 2232.3 nm is outside of the spectral coverage of these observations. These lines may be from molecules.

The strongest line is located at 2231.7 nm and has a maximum equivalent width in the umbra of about 3 mA. Two fainter lines lie at 2230.67 and 2230.77 nm, and each have maximum equivalent widths of about 1.5 mÅ. Each line shows a monotonically increasing equivalent width from about 5000 K down to the coldest measured temperatures of about 3700 K. Using the temperature behavior of various molecules described by WHL suggests that these lines behave most like the OH molecule; however with such small equivalent widths identification is difficult.

The weakest of the four lines is located at 2232.21 nm and has a maximum equivalent width near 0.8 mA. This line seems to have a reduced equivalent width in the umbra. A plot of the equivalent width versus temperature indicates a peak equivalent width at about 4500 K which is consistent with the behavior of the CN molecular lines.

Acknowledgements

W.C. gratefully acknowledges the support of the K.C. Wong Education Foundation, Hong Kong and the National Natural Science Foundation of China through award 10103004, and NSF of Yunnan through award 2001A0027Q during his visit to the National Solar Observatory in Tucson. Thanks go to C. Plymate and E. Galayda for immeasurable help at the telescope during the observations. The CSUN-NSO camera was purchased using the AFOSR award F49620-00-1-0355 and travel support was made possible through NSF ATM-0077624. M.J.P. thanks Carl Henney for a thorough review and helpful suggestions.

References

Jones, H. P.: 1999, *Bull. Am. Astron. Soc.* **31**, 958.

Kopp, G. and Rabin, D.: 1992, *Solar Phys.* **141**, 253.

Kuhn, J. R., Balasubramaniam, K., Kopp, G., Penn, M., Dombard, A., and Lin, H.: 1994, *Solar Phys.* **153**, 143.

Leka, K. D., Canfield, R. C., McClymont, A. N., and van Driel-Gesztelyi, L.: 1996, *Astrophys. J.* **462**, 547.

Livingston, W.: 2002, *Solar Phys.* **207**, 41.

Pahlke, K. D. and Wiehr, E. : 1990, *Astron. Astrophys.* **228,** 246.

Penn, M. J., Walton, S. R., Chapman, G., Ceja, J. A., and Plick, W.: 2003, *Solar Phys.*, **213,** 55.

Rüedi, I., Solanki, S. K., and Keller, C. U.: 1999, *Astron. Astrophys.* **348**, L37.

Rüedi, I., Solanki, S. K., Keller, C. U., and Frutiger, C.: 1998, *Astron. Astrophys.* **338**, 1089.

Sakurai, T.: 2001, in M. Sigwarth (ed.), *ASP Conf. Ser. 236*, Advanced Solar Polarimetry: Theory, Observation and Instrumentation, ASP, San Francisco, p. 535.

Vernazza, J. E., Avrett, E. H., and Loeser, R.: 1976, *Astrophys. J. Suppl. Ser.* **30**, 1.

Wallace, L., Hinkle, K., and Livingston, W.: 2001, 'Sunspot Umbral Spectra in the Region 4000 to 8640 cm^{-1} (1.16 to 2.50 μm)', N.S.O. Technical Report #01-001, Tucson, AZ.

Wiehr, E.: 1995, *Astron. Astrophys.* **298**, L17.